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Numerical Simulation of Free-Surface Liquid Lithium Flows in Tokamak Toroidal Geometry

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BACKGROUND

A LIQUID LITHIUM WALLS ELECTROMAGNETIC CONFINEMENT conceptual idea was presented at first APEX meeting:

CONFINE A THICK (1+ METER) FLOWING LIQUID LITHIUM LAYER BY ELECTROMAGNETIC AND CONTACT FORCES TO ENCLOSE A TOROIDAL MAGNETICALLY CONFINED PLASMA.

This is analogous to an extra TF coil turn, but of flowing liquid lithium.

Two axisymmetric streams of liquid lithium enter top of toroidal chamber.

The two streams are electrically insulated from each other at the top. Poloidal current injected via electrodes is driven through the streams which meet at the bottom of the chamber.

The resulting J X B forces push the streams against the chamber walls away from the plasma.

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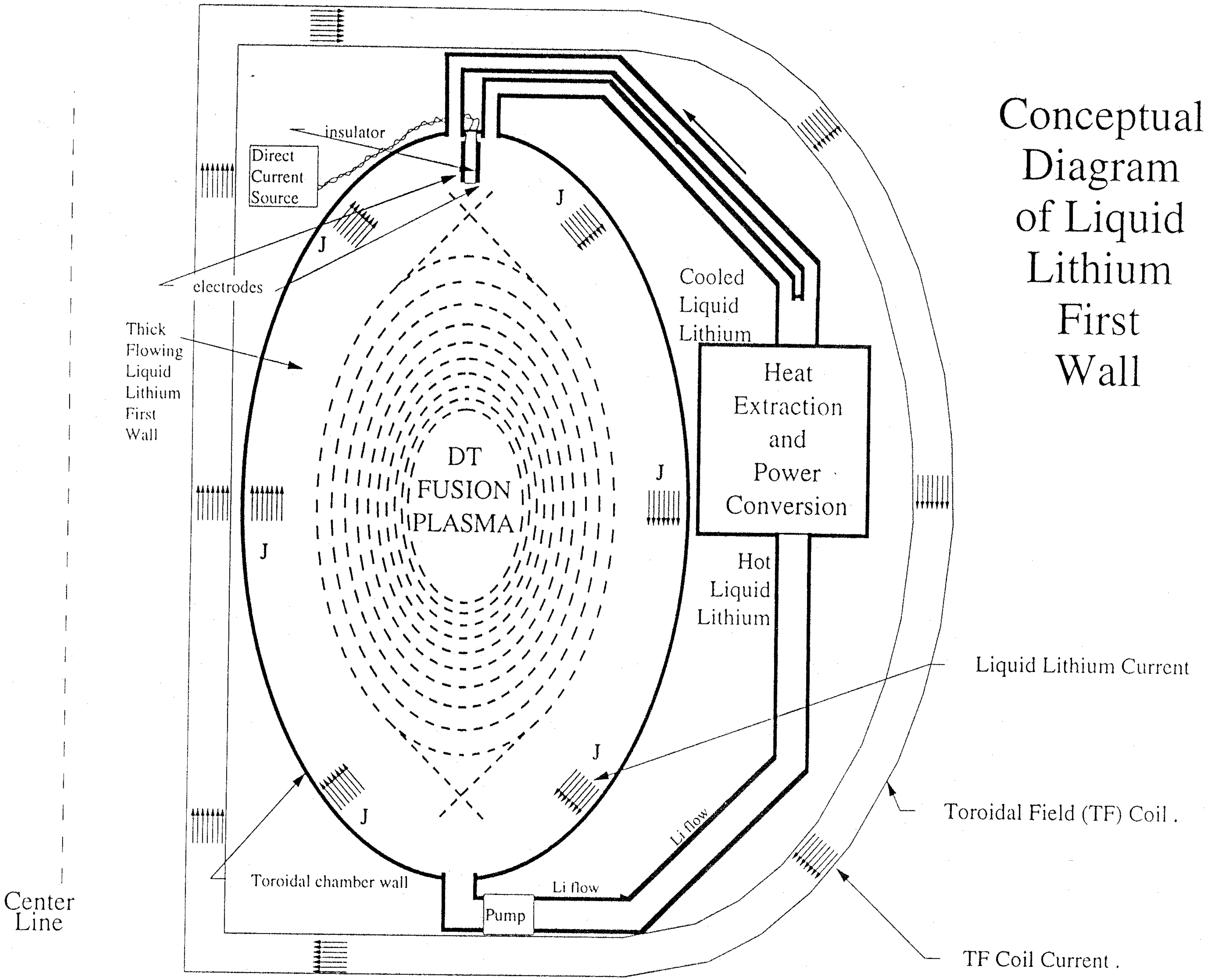
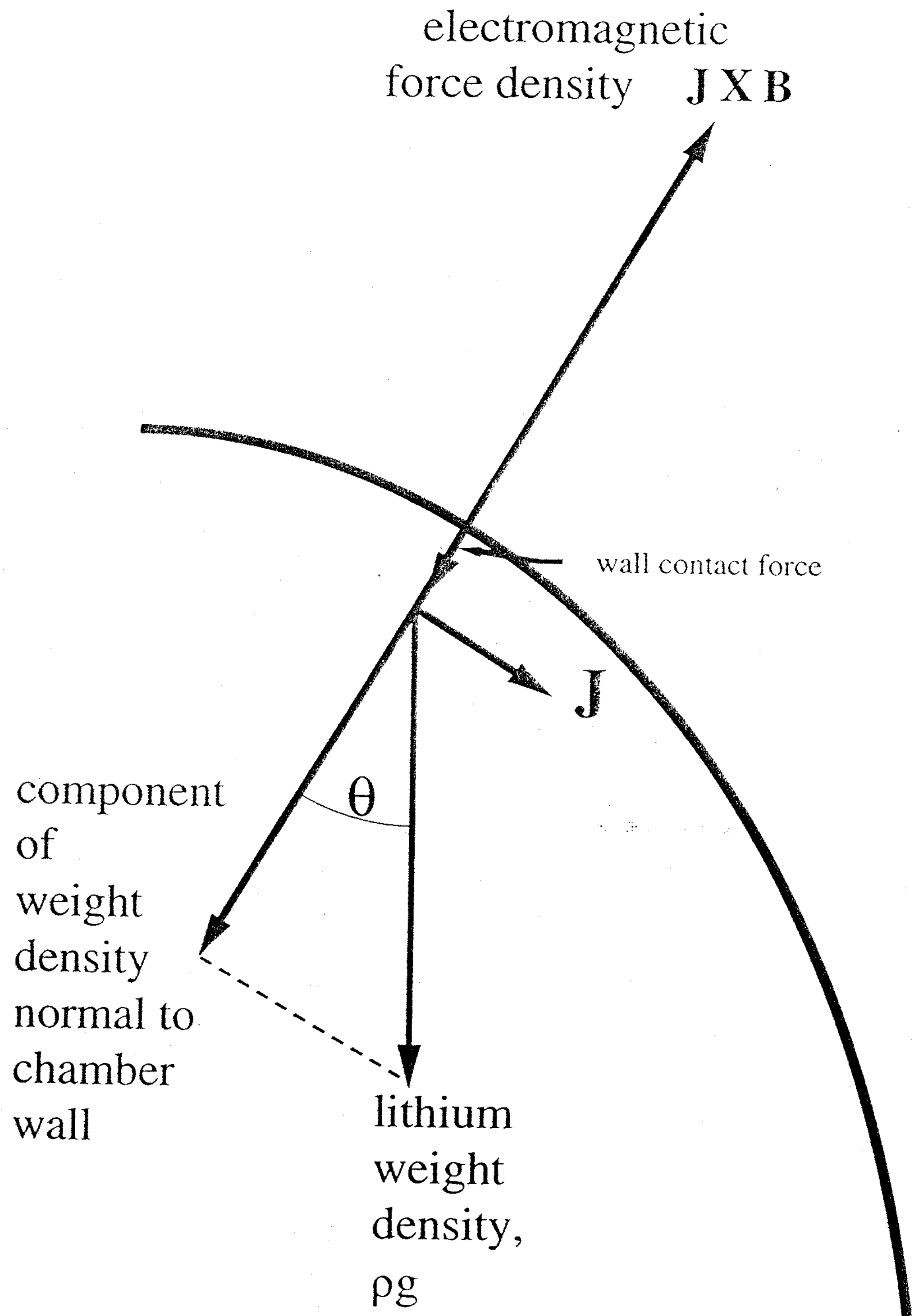
Conceptual Diagram of Liquid Lithium First Wall



Force Diagram for Liquid Lithium First Wall



<u>Wall Confinement Criterion:</u>

Wall Contact Force > 0 , i.e.

$$|\mathbf{J} \times \mathbf{B}| > \rho g \cos\theta$$

Confinement is guaranteed if

J (amperes/square meter) > 5047/B (Tesla)

CLAIMED FLOW FEATURES

Free surface in vacuum is assumed INSULATING.

FLOW PATH and INJECTED CURRENT follow FLUX SURFACE.

$B = B_P + B_T$

V parallel to B_P

J ~~Parallel~~ $\perp$ to B_T

Thus, No Toroidal Component in V, in V X B, in E, in J, or in J X B.

$V \times B = V \times B_T$

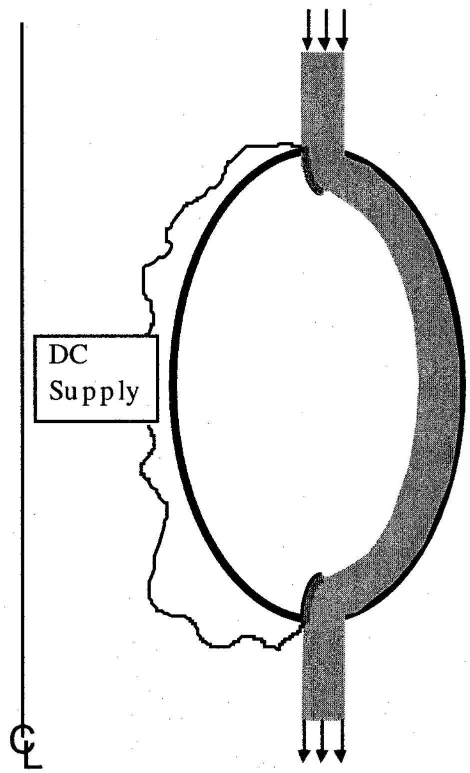
$J \times B = J \times B_T$

J X B force density is poloidal and perpendicular to V.

MHD effects on flow are minimized (since V & J are perpendicular to the direction of no variation and to the insulating open surface).

Simulation Need

A numerical simulation computer code is needed for engineering investigations of candidate AXISYMMETRIC, FREE SURFACE flows.



It should allow changing
-vessel and electrode geometry
-liquid inflow rate & pressure
-electrode voltage

It should calculate the free surface shape and the distributions of:
-electrical currents in the lithium and vessel wall
-magnetic field
-the liquid lithium's velocity field
-the liquid lithium's pressure field
-transit time, especially at surface

It should be economical to develop and use.

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SIMULATION DEVELOPMENT

The simulation code must be developed with minimum cost.

I have the need to use this simulation in order to properly investigate my proposal of massively liquid lithium walls, and I am developing it myself, as a part-time effort.

Existing software modules should be used if that saves effort. Alternatively, published successful numerical algorithms should be used.

I will try to have a working simulation before the next APEX meeting.

It will be necessary to develop a scheme for validating the simulation.

The simulation can also be used to investigate massively liquid lithium schemes for other axisymmetric confinement fusion reactors such as spheromaks, FRCs, RFPs.

Physics: Vector Equations & BCs

Incompressible	$\nabla \bullet \vec{\mathbf{u}} = 0$
Navier-Stokes:	$\dfrac{\partial \vec{\mathbf{u}}}{\partial t} + (\vec{\mathbf{u}} \bullet \nabla)\vec{\mathbf{u}} = -\dfrac{\nabla p}{\rho} + \nu \nabla^2 \vec{\mathbf{u}} + \vec{\mathbf{g}} + \dfrac{\vec{\mathbf{J}} \times \vec{\mathbf{B}}}{\rho}$
Liquid at Free Surface:	$p = 0$
Liquid at Material Walls:	$p > 0; \quad \vec{\mathbf{u}} = 0$
Ampere:	$\nabla \times \vec{\mathbf{B}} = \mu \vec{\mathbf{J}}$
Kirchoff:	$\Rightarrow \nabla \bullet \vec{\mathbf{J}} = 0$
	$\nabla \bullet \vec{\mathbf{B}} = 0$
Faraday:	$\nabla \times \vec{\mathbf{E}} = -\dfrac{\partial \vec{\mathbf{B}}}{\partial t}$
Ohm:	$\vec{\mathbf{J}} = \sigma(\vec{\mathbf{E}} + \vec{\mathbf{u}} \times \vec{\mathbf{B}})$
Vector Potential	$\nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$
Coulomb Gauge	$\nabla \bullet \vec{\mathbf{A}} = 0$
Induction & Electric Field:	$\vec{\mathbf{E}} = -\nabla V - \dfrac{\partial \vec{\mathbf{A}}}{\partial t}$
Electrode Voltages:	$V_1 = 0; \quad V_2 = V_{PS}$
At infinite distance:	$\vec{\mathbf{A}}, \vec{\mathbf{B}} \to 0$

Note: Driven currents in TF and PF electromagnets influence $\vec{\mathbf{A}}, \vec{\mathbf{B}}$.

Possible Difficulties and other Remarks

It is necessary simulate the unsteady nonlinear dynamics and integrate until reaching a steady-state.

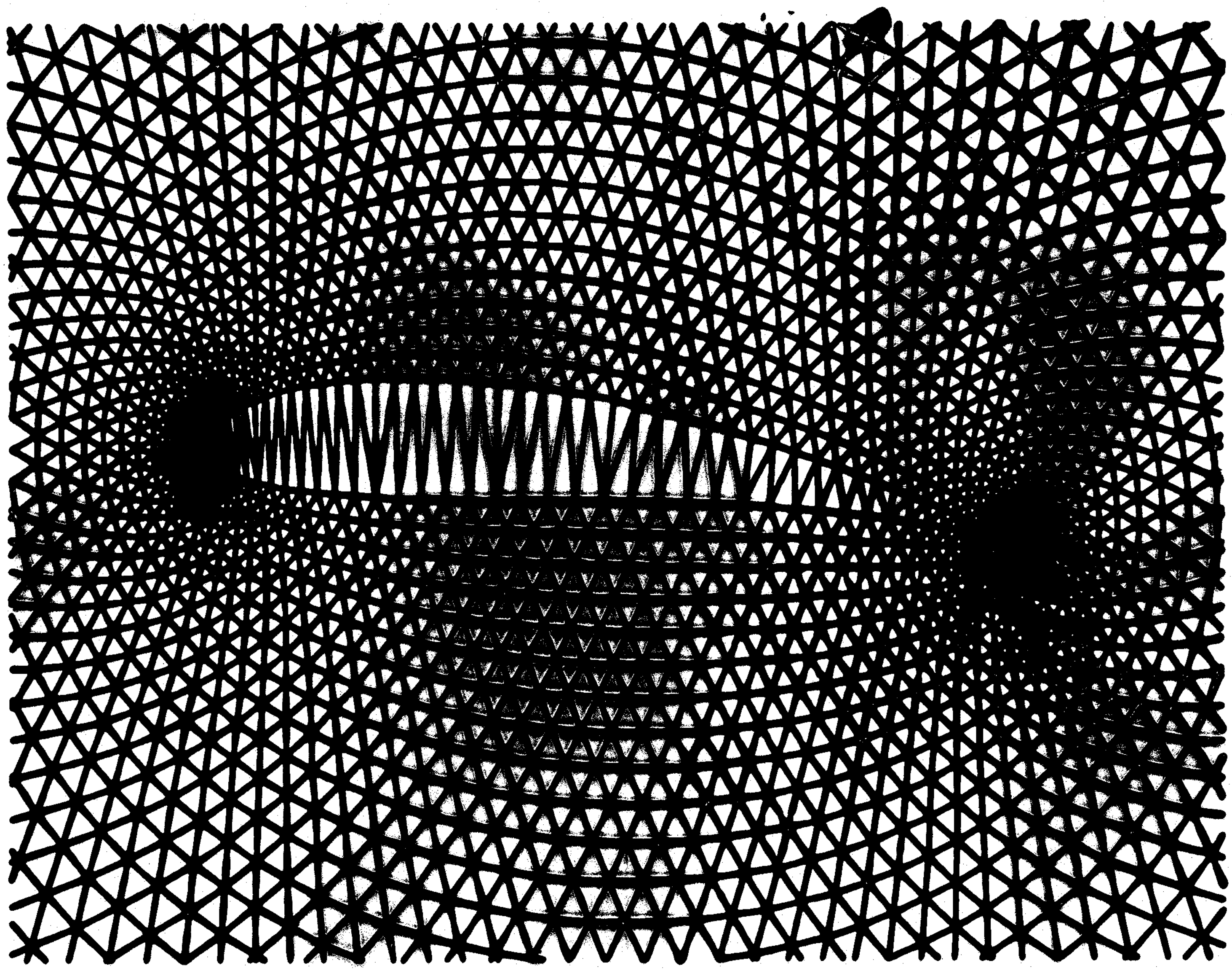
ANSYS was examined briefly. It may require too many modifications to be an effective starting point. No other existing codes were identified as a good starting point.

Galerkin FEM are directly applicable to these PDEs. But axisymmetric fluid flow simulations are less common than 2-D; errors are possible.

MHD dynamics will introduce numerical instability possibilities.

Important but extremely thin MHD boundary layers can form. A computational grid to resolve them must be capable of having a very small mesh size. Chosen solution will use an adaptive unstructured grid.

(I have coded Delauney triangulation of an unstructured grid via Bowyer's algorithm, using quadtrees and linked list data structures for high speed.)



REMARKS (CONT.)

A free surface is difficult to model. It's representation may be simplified via adaptive gridpoints which mark and move with the free surface. (Zienkiewicz's FEM text p624 describes such a time-stepping method.)

Incompressible flow is difficult to simulate. Approaches with a published successful track record include:

(1)Simulating compressibility. (This requires extremely tiny time steps to avoid numerical instabilities due to acoustic waves.)

(2)The Vorticity-Steam Function method (Never used with free surfaces.)

(3)The Penalty Function Method. (Requires subtle choices of pressure and velocity finite element interpolation methods to avoid"locking", .)

(4)Projection Method(s) (Problems unknown. Seems intuitive.)

My initial plans are to try using the Projection Methods.

Projection Methods

These have been successfully applied without MHD coupling (J X B = 0). The following published algorithm uses explicit Euler integration.

Step 0: Determine PPE boundary conditions:

At Material Surface: $\frac{\partial p^n}{\partial n} = \rho\{\nu\nabla^2\vec{\mathbf{u}}_n^n - (\vec{\mathbf{u}}^n \bullet \nabla)\vec{\mathbf{u}}_n^n\}$ [Propose $p^n = 0$ @ FS]

Step 1: Calculate "intermediate velocity field"

$$\tilde{\mathbf{u}} = \vec{\mathbf{u}}^n + (\Delta t)[-\nu\nabla^2\vec{\mathbf{u}}^n + (\vec{\mathbf{u}}^n \bullet \nabla)\vec{\mathbf{u}}^n]$$

[Propose including J X B here]

Step 2: Solve the "pressure Poisson equation" (PPE)

$$\nabla^2 p^n = \left(\frac{\rho}{\Delta t}\right)\nabla \bullet \tilde{\mathbf{u}}$$

(with PPE BCs)

Step 3: Correct the "intermediate velocity field"

$$\vec{\mathbf{u}}^{n+1} = \tilde{\mathbf{u}} - \left(\frac{\Delta t}{\rho}\right)(\nabla p^n - \rho\vec{\mathbf{g}})$$

Repeat for next time step.

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CONCLUSION

I will try to have a working simulation before the next APEX meeting.

Your remarks and advice on this effort would be most welcome.