

Modeling of liquid walls in APEX study

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Abstract. Liquid wall concept has a significant place in the Advanced Power Extraction (APEX) study as a part of the US program on Nuclear Fusion. In the present paper, different approaches currently developed for modeling liquid wall magnetohydrodynamics and heat transfer in APEX have been presented. Turbulent flows of low-conductivity liquids, such as molten salts (Flibe), are analyzed using a two-equation turbulence model and Direct Numerical Simulation. The analysis of liquid metal flows under a strong reactor magnetic field is based on the magnetohydrodynamic equations for laminar flows.

1. Introduction

In fusion applications, liquid metals (LMs) are traditionally assumed to be the best working fluids. Due to their high electrical conductivity, LMs have strong magnetohydrodynamic (MHD) interaction, however turbulence is completely suppressed except of some particular situations. Along with LMs, molten salts (10^4 times poorer conductors) are being carefully studied as a practical candidate for fusion applications [1]. For example, a 2 cm thick flow of Flibe moving in the poloidal direction along the reactor First Wall (FW) is used in the APEX (Advanced Power Extraction) study, that is an exploration of innovative concepts for fusion chamber technology carried out in the US since 1998 [1]. Unlike LMs, such flows do not experience significant MHD forces and remain turbulent. However, under a reactor strong magnetic field, turbulence pulsation in low-conductivity fluids can be partially suppressed with an accompanying reduction in heat transfer. Because of the significant differences in the behavior of LMs and low-conductivity fluids, different approaches are needed. In the APEX study, “K- ϵ ” two-equation turbulence model extended to MHD flows and Direct Numerical Simulation (DNS) are used for modeling of low-conductivity fluids. Laminar models based on the Navier-Stokes-Maxwell equations are applied to the analysis of LM flows. All approaches require the induced currents and the free surface location to be calculated simultaneously with other flow quantities. The paper describes briefly the approaches currently developed in the APEX study and gives some illustrations under conditions relevant to particular fusion designs. All details have not been shown here because of the paper limitations. More detailed analyses were presented in [1–5]. Unlike previous numerical studies, in which only relatively simple cases for laminar MHD flows in open channels were considered (developing flows in a one component constant magnetic field [6], fully developed flows in a two-component constant magnetic field [7]), the present ones introduce much more details, with more emphasize on MHD turbulence effects, and those related to multi-component and space-varying reactor magnetic fields.

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2. Low-conductivity fluid modeling using two-equation model

“K- ε ” turbulence model extended to MHD free surface flows has been used. The model is restricted to low magnetic Reynolds numbers. After applying Reynolds averaging to the Navier-Stokes-Maxwell equations with the conventional closure approximations, one can derive equations for the turbulent kinetic energy, K , and the dissipation rate, ε :

$$\frac{\partial K}{\partial t} + \langle \nu_j \rangle \frac{\partial K}{\partial x_j} = \underbrace{\nu_t \left(\frac{\partial \langle \nu_i \rangle}{\partial x_j} \right)^2}_{\text{Production}} + \underbrace{\frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_K} \right) \frac{\partial K}{\partial x_j} \right]}_{\text{Diffusion}} - \underbrace{\varepsilon - \varepsilon_{em}^K}_{\text{Dissipation}}; \quad (1)$$

$$\frac{\partial \varepsilon}{\partial t} + \langle \nu_j \rangle \frac{\partial \varepsilon}{\partial x_j} = C_1 \frac{\varepsilon}{K} \nu_t \left(\frac{\partial \langle \nu_i \rangle}{\partial x_j} \right)^2 + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] - C_2 \frac{\varepsilon}{K} \varepsilon - \varepsilon_{em}^\varepsilon. \quad (2)$$

All terms and coefficients in (1–2) are standard except of ε_{em} , which stands for the Joule dissipation. If the liquid flows in the poloidal direction, the wall-normal and the toroidal (streamwise) fields will have the strongest effect on the flow. The closures for the electromagnetic terms for these two field orientations, have been obtained as follows [2]:

$$\varepsilon_{em}^K = 1.9 \exp\{-1.0N\} \frac{\sigma}{\rho} B_0^2 K; \quad \varepsilon_{em}^\varepsilon = 1.9 \exp\{-2.0N\} \frac{\sigma}{\rho} B_0^2 \varepsilon. \quad (3)$$

The Stuart number, N , is built through $2h_0$ (h_0 is the characteristic flow thickness); σ is the electrical conductivity; ρ is the density, and B_0 is the applied magnetic field. The boundary conditions on K and ε at the free surface extend the standard ones by taking into consideration the effect of the magnetic field on the dissipation length scale. When modeling turbulent heat transfer, two effects were incorporated. The first one is the turbulence reduction by a magnetic field, and the second one is the turbulence redistribution in the near-surface region. Both effects are taken into account through the extension of the Fourier law as follows:

$$\langle t' \nu' \rangle = - \frac{\nu_t}{Pr_t} \frac{\partial T}{\partial y}. \quad (4)$$

The turbulent viscosity, ν_t , is defined in a conventional way by the Kolmogorov-Prandtl relation, while the turbulent Prandtl number, Pr_t , was evaluated using experimental data for open channel flows [8] as follows:

$$Pr_t = 0.7 \times [1 + \exp\{37 \times (y/h - 0.89)\}]. \quad (5)$$

Using the turbulence model (1–5) along with the mean flow equations and boundary conditions, Flibe flows with a free surface exposed to a high-density heat flux along the outboard FW were calculated under conditions related to the ARIES RS reactor (a high-power-density tokamak with an advanced, reversed shear plasma containment) for flows with axial symmetry. The initial thickness of 2.3 cm with the initial velocity of 10 m/s used in the CLIFF design (a part of the APEX study, which stands for the Convective Liquid Flow First Wall) results in almost uniform flow. Thick flows experience significant contraction due to gravity (Fig. 1). The surface temperature rise in CLIFF calculated with a heat flux of 2 MW/m² is about 80 K at the end of the 8 m FW section (Fig. 2). As calculations showed, in liquid walls thinner than 2 cm, the MHD effects will be almost negligible. For thicker walls, MHD effects on both mean flow and turbulence become pronounced.

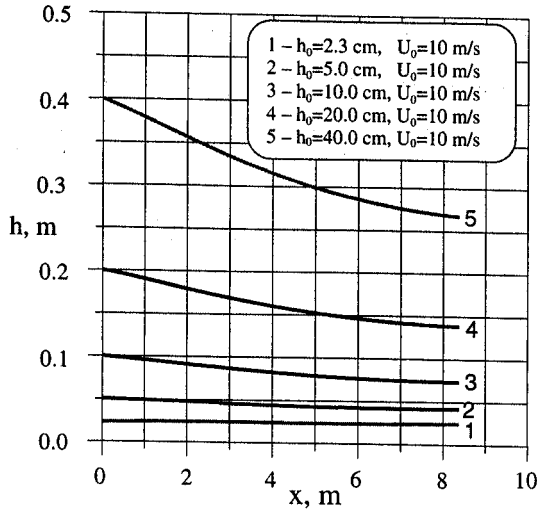


Fig. 1. Downstream variations of the thickness of the thick and thin Flibe liquid walls flowing over ARIES RS fusion reactor outboard FW under the influence of MHD forces and gravity. The liquid enters the chamber at the top and then flows in the poloidal direction to its bottom. Coordinate x measures the distance from the flow inlet along the FW in the main flow direction. Calculated with the present $K - \epsilon$ MHD turbulence model.

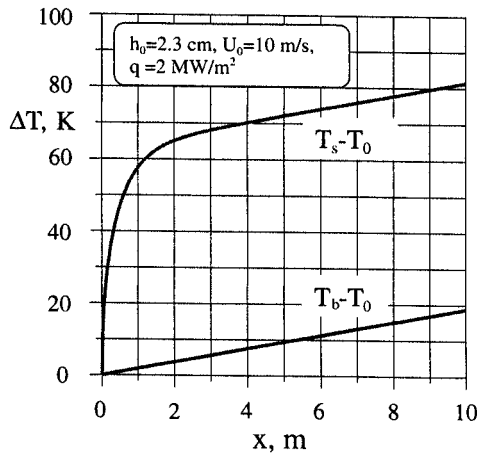


Fig. 2. Surface (T_s) and bulk (T_b) temperature rise in the thin Flibe liquid wall (ARIES RS, ClIFF) exposed to a high density uniform surface heat flux. T_0 is the temperature in the flow inlet. As it can be seen, the surface temperature grows fast within the entry section and then changes linearly within the thermally fully developed segment. Calculated with the present $K - \epsilon$ model.

3. Direct numerical simulation of open channel flows under a magnetic field

When developing the closures in the $K - \epsilon$ model, DNS calculations were used. In the present study, because of the limitations on the DNS technique, the simulations were restricted to open channel MHD flows with a non-wavy flat free surface. The Reynolds number based on the bulk velocity and the flow height was 2300. The magnetic field was applied in the streamwise ($Ha = 20.0, 30.0$) or in the spanwise direction ($Ha = 5.0, 10.0$). In terms of the tokamak vacuum chamber, spanwise and streamwise stand for the toroidal and poloidal directions respectively. The number of computational grids used in this

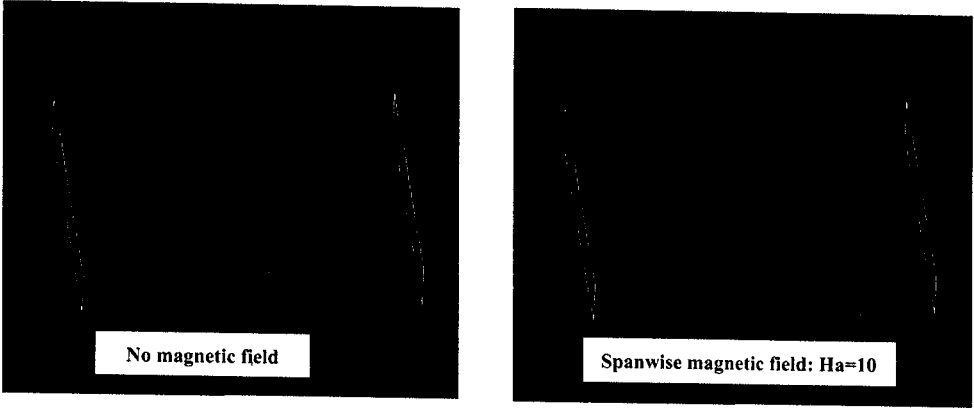


Fig. 3. Turbulence structures in open channel flow with and without a magnetic field: low-pressure regions, $p^+ < -3$, (grey); low-speed streaky strike structures, $u^+ < -3$, (blue). DNS for $Re_t = 150$. The turbulence structures are significantly reduced in the presence of a magnetic field.

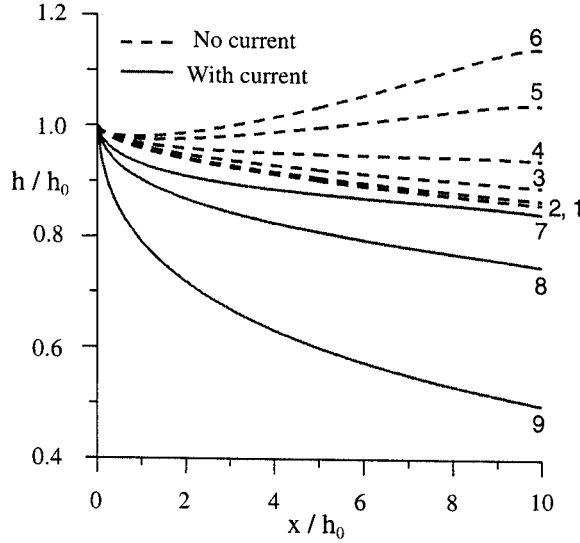


Fig. 4. Downstream flow thickness variations in open surface lithium flows with axial symmetry imposed to a linearly varying spanwise magnetic field with and without an applied electric current (j). Calculated with the present model (6-14). In the calculations, the flow length, L , is 20 cm, the initial flow thickness, h_0 , is 0.02 m, the initial flow velocity, U_0 , is 5 m/s. The liquid is propelled by applying a current. 1: $\Delta B_z/L = 0, j = 0$; 2: $\Delta B_z/L = 1.0 \text{ T/m}, j = 0$; 3: $\Delta B_z/L = 2.0 \text{ T/m}, j = 0$; 4: $\Delta B_z/L = 3.0 \text{ T/m}, j = 0$; 5: $\Delta B_z/L = 4.0 \text{ T/m}, j = 0$; 6: $\Delta B_z/L = 4.5 \text{ T/m}, j = 0$; 7: $\Delta B_z/L = 4.5 \text{ T/m}, j = 4.0 \text{ kA/m}^2$; 8: $\Delta B_z/L = 4.5 \text{ T/m}, j = 8.0 \text{ kA/m}^2$; 9: $\Delta B_z/L = 4.5 \text{ T/m}, j = 40.0 \text{ kA/m}^2$.

study was $256 \times 128 \times 128$ (streamwise, wall-normal, and spanwise). The electromagnetic part of the governing equations included the equation for the electric potential and Ohm's law. The turbulence structures, the low-speed streaky structures and the low-pressure regions, are shown in Fig. 3. Both spanwise and streamwise magnetic fields lead to turbulence suppression as it is evidenced by a reduced number of the turbulence structures, however the effect is much stronger in a spanwise field, which directly affects the fluctuating velocity field through the Lorentz force. The turbulence suppression by a streamwise field occurs indirectly through changing the pressure field. A magnetic field results in

$$j_\eta = -\frac{1}{\mu_0} \frac{1}{r} \frac{\partial}{\partial \xi} B'_\phi r; \quad (12)$$

$$j_\phi = \sigma (UB_\eta^0 - VB_\xi^0); \quad (13)$$

$$0 = \frac{1}{\sigma \mu_0} \left[\frac{1}{r} \frac{\partial}{\partial \xi} \left(r \frac{\partial B'_\phi}{\partial \xi} \right) + \frac{\partial^2 B'_\phi}{\partial \eta^2} \right] + \frac{\partial}{\partial \xi} (WB_\xi^0 - UB_\phi^0) - \frac{\partial}{\partial \eta} (VB_\phi^0 - WB_\eta^0). \quad (14)$$

Equations (6)–(8) are the projections of the momentum equation on the coordinate axes in the boundary layer approximation. Equation (9) is the mass conservation equation and Eq. (10) is the continuity equation for the electric current. Relations (11–13) are the expressions for the current components, and Eq. (14) is the induction equation. The notation used is standard. Symbols g_ξ and g_η stand for the acceleration due to gravity along with centrifugal or centripetal accelerations due to the motion of the liquid over the curved wall; $r(\xi)$ denotes the distance from the chamber axis to a given point in the liquid.

As the numerical analysis based on the finite-difference solution of (6–14) shows, the most significant MHD effects are caused by the wall-normal field and the spanwise space-varying magnetic field (both fields lead to an extra MHD drag and flow thickening). However in the presence of a gradient spanwise magnetic field, an excessive MHD drag can be overcome by applying a streamwise electric current that results in the “magnetic propulsion effect” [9]. Figure 4 illustrates the effectiveness of the magnetic propulsion by the example of a Lithium flow over a 20-cm length section. The flow becomes thicker as the spanwise magnetic field gradient grows because of the flow opposing effect caused by the induced electric currents. Applying an electric current creates a pressure gradient that accelerates the flow and finally makes it thinner.

5. Concluding remarks

The models and approaches described in the present paper can be applied to low-conductivity fluid turbulent MHD flows and laminar LM flows in channels with open surface. The application of these models to the liquid walls under conditions relevant to a fusion reactor reveals a variety of different MHD effects caused by a multi-component space-varying magnetic field. The detailed analysis of these effects is the essential part of the design work. An accurate mathematical model has been derived for open channel LM flows with axial symmetry. This model includes all three components of the applied magnetic field and all velocity components. The flows with no axial symmetry (for example, the axial symmetry will be broken if the flow along the FW is divided into sections by poloidal partitions) are essentially 3-D. In the presence of the partitions, the most crucial effect is that caused by a space-varying wall-normal magnetic field. As preliminary considerations show, changing the wall-normal magnetic field over the flow length gives rise to an axial potential difference within each section, which drives currents in the axial direction. These currents, while interacting with the magnetic field, produce forces driving the liquid in the toroidal direction from the core to the partitions that may lead to spilling the liquid over the partitions. This also results in a higher MHD drag than that in a uniform magnetic field. At present, a detailed 3-D numerical analysis of such flows under strong complex tokamak magnetic fields can be hardly performed. However, a number of simplified models has been derived. Many illustrations can be found on the official APEX site (www.fusion.ucla.edu/APEX). Future studies will concentrate on both the design options and further improvements of the models and computer codes.

reducing turbulence in the bulk and suppressing its generation near the wall. Some features showing transition to 2-D turbulence can be seen, such as enlargement of vortices (inverse energy cascade). However, stretching vortices along the magnetic field lines typical for developed 2-D MHD turbulence is not observed probably because of the low turbulence Reynolds number regime ($Re_t = 150$) realized in the present DNS calculations.

4. Modeling LM flows with axial symmetry

As examples of MHD flows with axial symmetry we refer to the lithium flow over the central column in the NSTX reactor¹ and outboard FW flow in the already mentioned ARIES RS reactor. When developing a mathematical model, the flow was assumed to be thin. It means the flow thickness is much smaller than the characteristic dimensions of the vacuum chamber and both main radii of curvature of the structural wall. In this approximation, only streamwise variations of the external magnetic field are important. Besides that the diffusion transport in the flow direction is negligible compared to that by convection. The second assumption is the inductionless approximation that implies the magnetic Reynolds number is much smaller than unity. Under this condition, the induced magnetic field, B' , is much less than the external one, B^0 , and can be neglected where they appear together. We also assume the external magnetic field does not vary in time. Third, in a strong "reactor-type" magnetic field, the LM flow will be laminarized, so that introduction of turbulence effects in the model is not needed. The governing equations were formulated in boundary fitted coordinates. In what follows " ξ ", " η ", and " ϕ " are used for the streamwise (poloidal), wall-normal, and spanwise (toroidal) coordinates respectively. Omitting details, the equations have been derived in terms of the velocity components and the induced magnetic field as follows:

$$U \frac{\partial U}{\partial \xi} + V \frac{\partial U}{\partial \eta} = -\frac{1}{\rho} \frac{\partial P}{\partial \xi} + \nu \frac{\partial^2 U}{\partial \eta^2} + g_\xi + \frac{1}{\rho} (j_\eta B_\phi^0 - j_\phi B_\eta^0); \quad (6)$$

$$0 = -\frac{1}{\rho} \frac{\partial P}{\partial \eta} + g_\eta + \frac{1}{\rho} (j_\phi B_\xi^0 - j_\xi B_\phi^0); \quad (7)$$

$$U \frac{\partial W}{\partial \xi} + V \frac{\partial W}{\partial \eta} = \nu \frac{\partial^2 W}{\partial \eta^2} + \frac{1}{\rho} (j_\xi B_\eta^0 - j_\eta B_\xi^0); \quad (8)$$

$$\frac{1}{r} \frac{\partial U r}{\partial \xi} + \frac{\partial V}{\partial \eta} = 0; \quad (9)$$

$$\frac{1}{r} \frac{\partial j_\xi r}{\partial \xi} + \frac{\partial j_\eta}{\partial \eta} = 0; \quad (10)$$

$$j_\xi = \frac{1}{\mu_0} \frac{\partial B'_\phi}{\partial \eta}; \quad (11)$$

¹The National Spherical Torus Experiment (NSTX) is a fusion device constructed by the Princeton Plasma Physics Lab.

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