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ANALYSIS OF MHD EFFECTS IN ENTRANCE REGIONS OF SELF-COOLED LIQUID METAL BLANKETS

Kathryn A. Snider and Mohamed A. Abdou
6288 Boelter Hall
University of California, Los Angeles
Los Angeles, CA 90024
(213) 206-0501

Abstract

A key problem in self-cooled liquid metal blankets for magnetic fusion reactors is the high pressure drop due to MHD effects. This work is concerned with analysis of the contribution to the pressure drop from longitudinal eddy currents for flow in a transverse magnetic field which varies in the direction of the flow. The order of magnitude of these currents is, in some cases, comparable to eddy currents generated in the plane perpendicular to the flow. In magnetic fusion reactor blankets the liquid metal must flow from regions with low or no magnetic fields to regions of higher magnetic fields.

A two-dimensional finite difference computer code was developed to determine longitudinal eddy current distributions. Pressure drops are calculated using these distributions. Results are compared with the presently limited experimental data available and with theoretical predictions. Estimates of uncertainties in calculating pressure drops are obtained. These uncertainties are found to be large and indicate a need for further study.

1. Introduction

Liquid metal MHD flow has received a great deal of attention lately because of its potential application to the magnetic fusion reactor blanket. Single channel, straight duct flow in a uniform magnetic field is well understood. However, because of the complex geometry necessary in order to satisfy reactor configuration and heat transfer requirements, the fusion reactor blanket flow will deviate considerably from straight duct uniform field flow. A particular example of this is that the liquid metal must enter and exit the magnetic field since heat and tritium extraction will be done outside of the magnetic field.

Liquid metal flow differs from ordinary hydrodynamic flow in that when it flows across a transverse magnetic field, electrical eddy currents are generated in the plane perpendicular to the flow. These currents can interact with the magnetic field to induce a large pressure drop. At entrance and exit regions, a pressure drop occurs as a result of eddy current generation in the plane perpendicular to the magnetic field. This paper analyzes this pressure drop.

2. Background

When a conducting fluid flows in a direction perpendicular to a magnetic field, the free charges in the fluid feel a force proportional to q ($\underline{V} \times \underline{B}$), the Lorentz force, where q is the charge, V is the velocity, and B is the magnetic field. The Lorentz force induces charge separation, thus electrical eddy currents flow in the plane perpendicular to the direction of fluid flow. This is described by Ohm's law for moving media,

$$\underline{j} = \sigma(\underline{E} + \underline{V} \times \underline{B}) \qquad (1)$$

where j is the eddy current density, σ is the conductivity and E is the induced electric field. The eddy current path is partially dependent on whether the duct walls are conducting or nonconducting. The importance of the electrical eddy currents lies in their interaction with the magnetic field. This interaction produces a body force which results in a pressure gradient

$$\nabla p = \underline{j} \times \underline{B} \qquad (2)$$

In a duct with nonconducting walls, the net body force is zero

$$\iint dA\, \underline{j} \times \underline{B} = 0 \qquad (3)$$

where A is the duct area, because all the current must return in the fluid. In a conducting duct, however, some of the current can return through the walls, thus there is a net body force which results in a pressure drop.

In regions of non-uniform magnetic field, electrical eddy currents are generated in an additional manner. Because the product $\underline{V} \times \underline{B}$ changes in a streamwise direction, the magnitude of the charge separation changes. Thus, the induced potential varies longitudinally, causing eddy currents to flow in the plane perpendicular to the B field (Figure 1). There will be a net body force due to these longitudinal currents whether the duct walls are conducting or nonconducting.

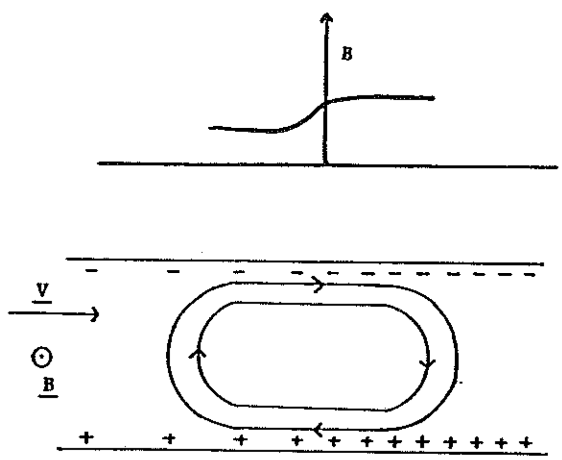

Fig. 1: Induced Eddy Currents

When a non-uniform transverse magnetic field is present, the electromotive force will be in such a direction as to cause a positive pressure gradient at one end of the current loop, and a negative pressure gradient at the other end, accelerating and decelerating the flow (Figure 2). The net effect, however, is negative because the positive pressure gradient occurs in the region where the magnitude of the magnetic field is less than that in the region with the negative pressure gradient.

Entrance and exit effects are a special case of non-uniform B field flow. In this case the eddy currents may return in regions with little or no magnetic field. There has not been much study of the entrance and exit pressure drop. One calculation (Ref. 1)

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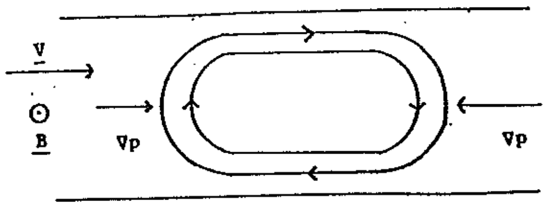

Fig. 2: Pressure Gradients

yielded an equation of the form

$$\Delta p = \kappa_1 \sigma a V B^2 \tag{4}$$

where a is the duct half width. A second study (Ref. 2) gives an equation of the form

$$\Delta p = \kappa_2 \sigma a v B^2 M^{-1/2} \tag{5}$$

where M is the Hartmann number, $M = Ba\sqrt{\sigma/\mu}$, μ is the fluid viscosity. The Hartmann number is the ratio of magnetic forces to viscous forces.

3. Description of Methodology

The following assumptions were made in solving this problem:
1. fluid velocity is constant;
2. velocity profile is slug;
3. induced magnetic field is negligible;
4. walls are nonconducting;
5. problem is 2-D.

The equations which must be solved in this case are

$$\underline{j} = \sigma(\underline{E} + \underline{V} \times \underline{B}) \tag{6}$$

where $\underline{E} = -\nabla\phi$

$$\nabla \cdot \underline{j} = 0 \tag{7}$$

where ϕ is the potential.

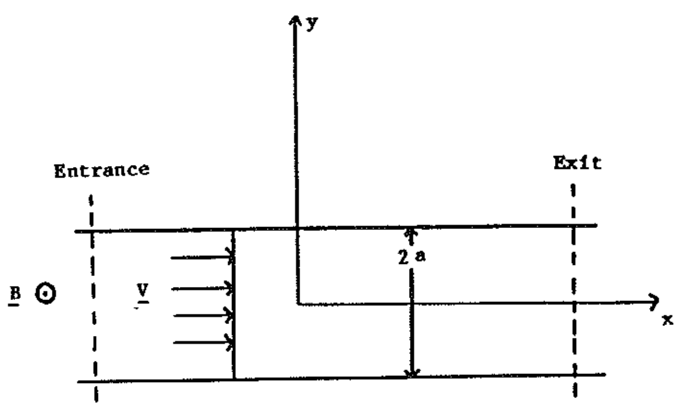

Fig. 3: Geometry of Problem

Figure 3 shows the geometry employed. In this 2-D problem, Eq. (6) and (7) reduce to

$$j_x = -\sigma \frac{\partial\phi}{\partial x} \tag{8}$$

$$j_y = \sigma\left(-\frac{\partial\phi}{\partial y} - V_x B_z\right) \tag{9}$$

$$\frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} = 0 \tag{10}$$

Equations (8) and (9) can be used in Equation (10) to yield

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{11}$$

the 2-D Laplace Equation. In a nonconducting duct, current cannot flow in the duct walls, thus the boundary conditions are

$$\frac{\partial\phi}{\partial y} = -V_x B_z \quad \text{at} \quad y = \pm a \tag{12}$$

Because of symmetry, the potential is constant at the duct centerline. If the potential is taken as zero there, the remaining boundary condition is

$$\phi \rightarrow 0 \quad \text{as} \quad x \text{ becomes large} \tag{13}$$

Therefore, there are no longitudinal eddy currents far upstream from the entrance and far downstream from the exit.

The liquid metal in the channel can be modeled as an electrical circuit. The electromotive force, VB, is represented by a battery, and the internal resistance of the fluid by a resistor (Figure 4). If there is a potential difference between nodal points, current will flow.

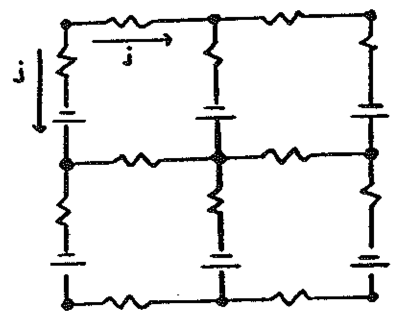

Fig. 4: Electric Circuit Model of Channel

A 2-D finite difference code was written to calculate the eddy current paths and resulting pressure drop.

The potential distribution was found by iterating on the finite difference form of Eq. (11) using successive overrelaxation. The currents were then solved for using Eqs. (8) and (9).

Figure 5 shows a typical eddy current distribution for the case where the magnetic field changes abruptly, i.e., goes from zero to some constant value B. The magnitude of the eddy current density is largest near the entrance area and decreases quickly.

Figure 6 shows a typical eddy current distribution for the case of a gradually increasing magnetic field. In this calculation, the field was assumed to increase linearly. Again, the eddy currents are centered around the entrance region. However, because the magnetic field increases gradually, some of the current returns in regions of magnetic field with magnitude less than that of the maximum magnetic field. It is logical to expect that the pressure drop will be less for a gradually increasing field than for an abruptly increasing one.

In order to determine the pressure drop, the Navier-Stokes equation is used.

$$\rho \frac{D\underline{V}}{Dt} + \nabla p = \eta \nabla^2 \underline{V} + \underline{j} \times \underline{B} \tag{14}$$

The first term represents velocity profile development, the second is the pressure gradient, the third is the

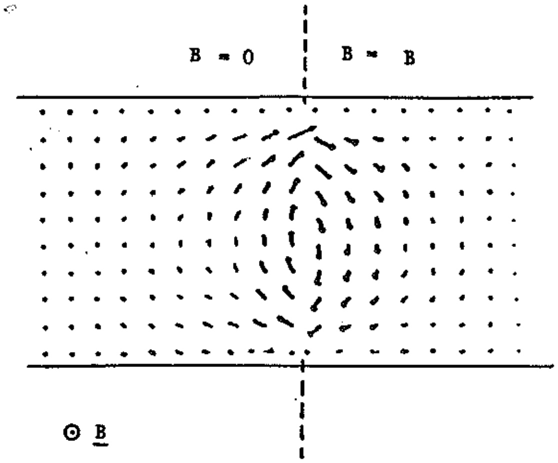

Fig. 5: Eddy Currents Due to Abrupt B

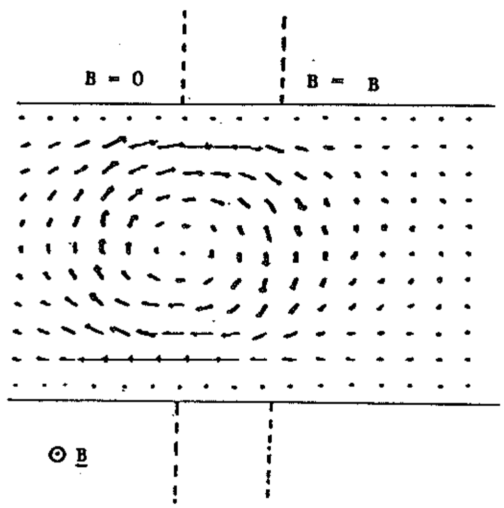

Fig. 6: Eddy Currents Due to Gradually Increasing B

viscous term, and the fourth represents the body force. Because the problem being solved was assumed to have constant velocity, this equation reduces to

$$\nabla p = \underline{j} \times \underline{B} \quad (15)$$

The pressure drop will therefore be

$$\Delta p = (\underline{j} \times \underline{B})\, \Delta x \quad (16)$$

where Δx is the length over which the longitudinal currents prevail.

The pressure drop was calculated for various channel half-widths and for various values of B. The results of these calculations were compared to the functional forms of references 1 and 2:

$$\Delta p = \kappa_1 \sigma a V B^2 \quad (17)$$

$$\Delta p = \kappa_2 \sigma a V B^2 M^{-1/2} \quad (18)$$

Our results fit equation (18). For an abruptly increasing magnetic field

$$\Delta p = 35.8\ \sigma a V B^2 M^{-1/2} \quad (19)$$

In the case of the gradually increasing magnetic field, the magnetic field was assumed to increase linearly from B = 0 to some maximum value. The length over which this increase takes place is called the fringe length. The value of the coefficient in equation (19) varies with the fringe length. Table 1 shows examples of these coefficients for a channel with a = 0.53. The coefficient decreases with increasing fringe length, as expected.

TABLE 1

Fringe Length	K_2
0	35.8
.02	24.9
.045	18.1
.08	12.4

The discrepancy between the functional forms of Eq. (17) and Eq. (18) is due to the different methods employed in each study. In Ref. 1, the same assumptions outlined in Section 3 were used to calculate the potential distribution. However, a new velocity profile was then calculated assuming the velocity deviates from the average velocity by a small amount i.e.,

$$V = V_m + w \quad (20)$$

where V_m is the mean velocity and w is the small deviation. Nonlinear terms in w were discarded, thus the Navier-Stokes equation could be written as

$$\rho V_m \frac{dw}{dZ} = j_y B - \frac{\partial p}{\partial Z} \quad (21)$$

The pressure drop is then

$$\frac{1}{a} \int_0^a dy \int_{-\infty}^{\infty} j_y B(x)\, dx + \frac{\rho V_m}{a} \int_0^a w dy \quad (22)$$

The pressure drop is found at y = a, where $j_y = 0$

$$\Delta p = \rho V_m w(a) = 0.27\ \sigma a V_m B^2 \quad (23)$$

The analysis of Ref. 2 is three-dimensional, taking into consideration velocity profile deformation and eddy current densities. Thus, it is not unexpected that Eq. (18) more accurately describes the pressure drop.

4. Comparison with Experiments

There have not been many experiments to assess the pressure drop at entrance and exit regions. One well-documented one (Ref. 3) found that

$$\Delta p = .062\ \sigma a V B^2 \quad (24)$$

This experiment was performed in order to determine the coefficient in Eq. (17), thus that functional form was assumed. Only one channel size was used, so any variation with a other than linear as Eq. (17) assumes, would not be noticed. In addition, these experiments were carried out for a very small range of B. Therefore, this does not rule out Eq. (18) as being the correct functional form.

If the results of the experiment are put in the form of Eq. (18), the result is

$$\Delta p = 11.2\ \sigma a V B^2 M^{-1/2} \quad (25)$$

It is not suprising that the coefficient in Eq. (25) is less than that in Eq. (19). Magnetic fields do not actually increase abruptly, and since the pressure drop decreases with increasing fringe length, experimental results should yield a coefficient less than that in Eq. (19).

Typical values in a fusion reactor are a = .35 m,

$\sigma = 3 \times 10^6\ (\Omega m)^{-1}$, $V = .12$ m/s, and $B = 4T$. The resulting pressure drop is $\Delta p = .22$ MPa. This is a small but significant pressure drop relative to the total pressure drop of 4-5 MPa expected in a fusion reactor blanket. However, channel walls in a fusion reactor will probably not be non-conducting, so it is likely that the actual pressure drop will be somewhat higher than that predicted by Eq. (19). The magnetic field will not increase abruptly however, and this is an advantage in this type of pressure drop.

5. Conclusions

The pressure drop due to the exit and entrance regions in a magnetic confinement fusion reactor were found to be small. This calculation was done assuming non-conducting walls. Because the channels walls will probably be conducting, further planned work at UCLA includes modifying the code to handle conducting walls. In addition, an application of this work which could be done is to investigate velocity profile development. After calculating the potential distribution, a new velocity profile could be calculated from the Navier-Stokes equation. This new velocity profile can be used to calculate a new potential distribution. This iterative process would be repeated until velocity profile convergence is achieved. It is not known whether this problem will converge. Velocity profiles are also important to heat transfer in liquid metal blankets.

6. References

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