

DEVELOPMENT OF A COMPUTATIONAL METHOD FOR THE FULL SOLUTION OF MHD FLOW IN FUSION BLANKETS

Chang N. KIM, Ali H. HADID and Mohamed A. ABDOU

University of California, Los Angeles, Los Angeles, CA 90024, USA

An advanced numerical scheme based on the finite difference method is presented for the full solution of a steady, incompressible, viscous, and electrically conducting flow with relatively large Hartmann numbers and interaction parameters. The solution method solves the coupled Navier-Stokes and Maxwell's equations at a low magnetic Reynolds number, and focuses on predicting key performance parameters for the design of fusion liquid metal blankets. The computational algorithm includes the effects of advection and diffusion, and is intended as a full solution, which has the potential capability of treating unsteady flow and applicability to heat and mass transfer in MHD flows.

The present method uses primitive variables (velocity and pressure) for hydrodynamic variables and the electric potential, and employs a finite volume approach with a staggered grid system for fast convergence and physical accuracy. A pressure equation of the Poisson type is used for pressure-velocity-potential coupling.

Results for two-dimensional channel flows with a nonuniform magnetic field at a Hartmann number and an interaction parameter of about 10^3 are obtained. The extension of the present numerical scheme to a three-dimensional MHD flow has been carried out and some preliminary results of a straight duct flow with a rectangular cross-section are presented.

1. Introduction

The design of MHD devices, including the liquid metal blanket in a fusion reactor, requires the development of accurate and efficient methods for predicting MHD flows in a strong magnetic field. It has been shown [1] that there is a strong need for capabilities to predict MHD flow fields in fusion reactor blankets at relatively large values of M and N .

In a uniform magnetic field, Hunt [2] analyzed a fully developed flow in a rectangular duct. He obtained an M-shaped velocity profile having a large velocity near the insulating walls. Under a nonuniform magnetic field, Shercliff [3] noted that M-shaped velocity distributions were formed in the vicinity of the edges of the magnetic poles in a two-dimensional channel flow.

Many theoretical analyses exist which have been mainly concerned with flows at infinitely large values of M and N . Walker and Ludford [4], Walker [5], and Petrykowski and Walker [6] have employed asymptotic techniques to analyze MHD flows in rectangular and circular ducts with variable cross-sectional areas. These analyses consider the case where the values of M and N are extremely large, so the regions of interest are regarded as inviscid and inertialess. These methods are approximate and have limitations since the actual MHD

flow field in practical blankets would become very complicated due to the inertial forces and unsteady effects depending upon the flow condition and the geometry. Therefore, full MHD solutions are required which can take into account all the different forces acting on MHD flows.

While the Full Solution Approach has great advantages of predicting realistic flow fields, it suffers from numerical difficulties. For large values of M and N , the ratio of each of the inertial and viscous forces to the magnetic forces is small. These different orders of magnitude of the terms in the equation of motion leads to difficulties in obtaining a converged solution.

Some numerical methods have been employed for MHD flows in a simple geometry. The finite element methods were used by Singh and Lal [7], Abdou [8], and Winowich and Hughes [9] for one- and two-dimensional flows at Hartmann numbers of up to 100. Also, the finite difference method was employed by Ramos and Winowich [10] and Abdou [8] for two-dimensional flows at Hartmann numbers of up to 100.

This work is concerned with the development of an efficient algorithm for obtaining the full MHD solution for values of M and N for geometries relevant to blankets in magnetic fusion reactors. In the next section, nondimensional governing equations and numeri-

cal procedures are presented. These are followed by a presentation of the computed velocity and pressure distributions.

2. Problem formulation

MHD equations that describe the equations of motion and Maxwell's equations in a steady, incompressible, constant-property, electrically-conducting, laminar flow are well known. In most practical MHD applications, the induction equation is not treated since magnetic Reynolds numbers are negligible.

The variables for an MHD flow can be scaled as follows:

$$\begin{aligned} X &= \frac{X^*}{L_0}, & U &= \frac{U^*}{U_0}, & B &= \frac{B^*}{B_0}, \\ J &= \frac{J^*}{\sigma U_0 B_0}, & \Phi &= \frac{\Phi^*}{U_0 L_0 B_0}, & p &= \frac{p^*}{\rho_0 U_0^2 N}, \end{aligned} \quad (1)$$

where the starred variables are dimensional quantities.

From the above scalings, the resulting governing equations can be written in a nondimensional form as conservation of mass:

$$\nabla \cdot \mathbf{U} = 0; \quad (2)$$

equation of motion:

$$\frac{1}{N} \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla p + \mathbf{J} \times \mathbf{B} + \frac{1}{M^2} \nabla^2 \mathbf{U}; \quad (3)$$

Ohm's law:

$$\mathbf{J} = -\nabla \Phi + \mathbf{U} \times \mathbf{B}; \quad (4)$$

conservation of charge:

$$\nabla \cdot \mathbf{J} = 0. \quad (5)$$

The large difference in the sizes of the different forces in the equation of motion can be treated numerically by employing the control volume approach together with the staggered grid system so that integral conservations of the physical properties can be exactly satisfied over the control volumes.

In the present method, a pressure equation of the Poisson type is used which takes into account the effects of not only fluid velocities but electric currents on the pressure distributions. This pressure equation can be obtained by taking the divergence of eq. (3) together with eq. (2) as

$$\nabla^2 p = \frac{1}{N} \nabla \cdot (\mathbf{U} \cdot \nabla \mathbf{U}) - \nabla \cdot [(-\nabla \Phi + \mathbf{U} \times \mathbf{B}) \times \mathbf{B}]. \quad (6)$$

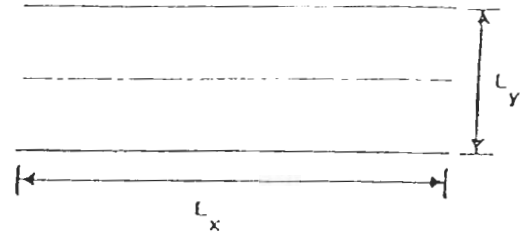


Fig. 1. Schematic of the MHD channel.

Also, the combination of eqs. (4) and (5) gives the Poisson type equation for the electric potential as

$$\nabla^2 \Phi = \nabla \cdot (\mathbf{U} \times \mathbf{B}). \quad (7)$$

A full implicit procedure is employed to solve eqs. (3), (6) and (7). Iterative solution steps are repeated until the residual values for velocities, pressure, and electric potential are below a tolerance level ($\epsilon < 10^{-3}$).

3. Results and discussion

In this section, some applications of the present solution scheme to two- and three-dimensional MHD flows under large values of M and N are presented.

3.1. A two-dimensional problem

A two-dimensional MHD flow in an infinite channel of length $L_x = 200$, height $L_y = 1$, with electrically insulating walls is considered as shown schematically in fig. 1. The nondimensional applied magnetic field, applied out of the channel plane, is indicated in fig. 2, which shows that the field has a concentrated part near $x = L_x/2$. The calculations reported here were performed for the two different cases: (1) $N = 100$, $M = 316$, $\text{Re} = 1000$ and (2) $N = 1000$, $M = 1000$, $\text{Re} = 1000$. The computational domain is the upper half of the

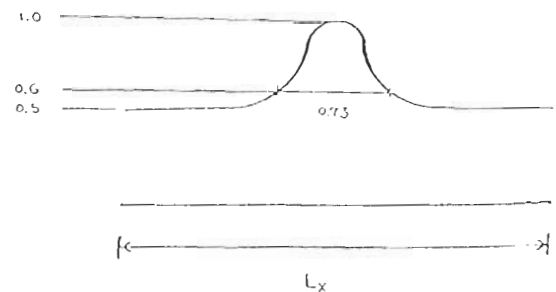


Fig. 2. Nondimensional applied magnetic field ($B_z = 0.5(1 + 1/\cosh[2\pi(x - L_x/2)])$).

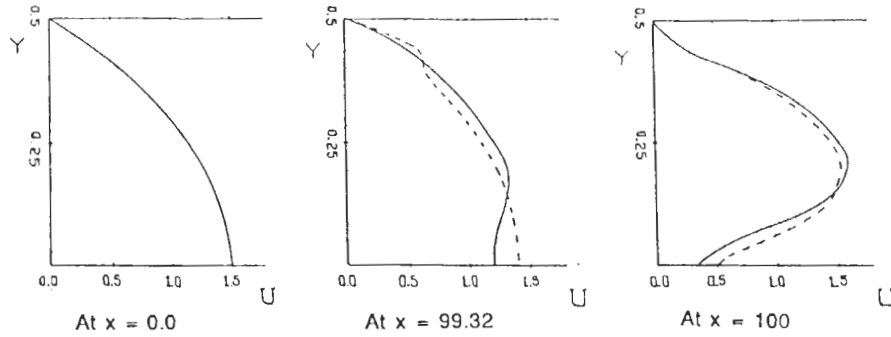


Fig. 3. *U*-velocities at different locations.

channel plane and a 93×17 unequally spaced grid system is used which concentrates the grid near the upper channel wall, centerline, and where the magnetic field is concentrated.

Fig. 3 shows the nondimensional axial velocity distributions at several selected axial locations, which illustrate the flow behavior from far upstream to the half span of the length of the channel. The figure also indicates that the axial velocity distribution for the second case at $x = 99.32$ has an M-shaped profile, while that for the first does not. But, at $x = 100$ the velocity profiles for the two cases have the M-shape characteristics.

In Fig. 4 the velocity distributions along the center of the channel are shown. The results in the figure show that the axial velocities become very small in the concentrated field region, which means that the axial velocities near the two plates become fast with the M-shaped profiles.

Fig. 5 shows the pressure distributions along the channel. Note the pressure pulses near the concentrated

field region which are caused by the interaction between the applied magnetic field and the electric currents in the transverse direction. This pressure pulse can cause large stresses on the channel walls.

An important observation can be made from comparing the results for the two cases which differ only in the magnitude of M and N . The solution obtained at $N = 100$ and $M = 316$ predicts qualitatively the same behavior and quantitatively to a reasonable approximation the solution obtained at $N = 1000$ and $M = 1000$. This indicates that reasonable approximations at higher M and N ($\sim 10^4 - 10^5$) can be obtained from the solution at M and N of $\sim 10^3$. This is an important observation in assessing the practicality of the full solution to fusion blankets.

3.2. A three-dimensional problem

A three-dimensional MHD flow in straight duct with a rectangular cross-section is considered with an applied magnetic field in the z -direction, as shown in fig. 6. For

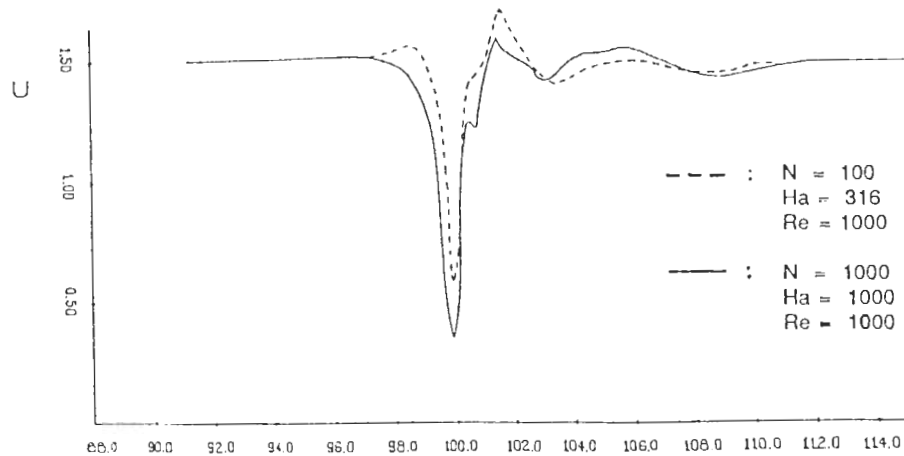


Fig. 4. The behavior of *U*-velocities along the centerline of the channel.

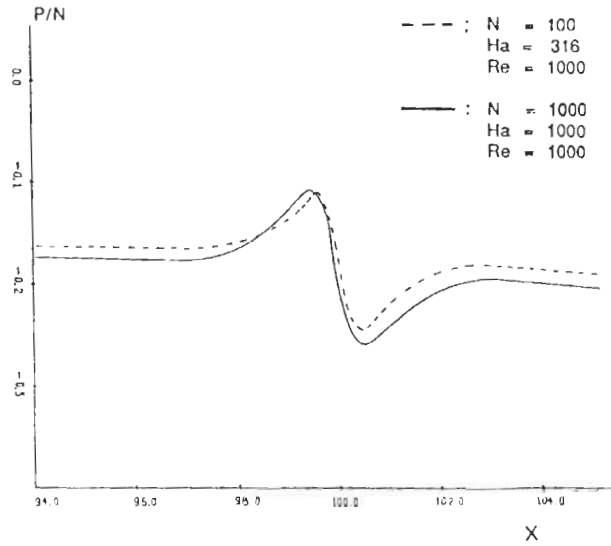


Fig. 5. Pressure distribution along the centerline of the channel.

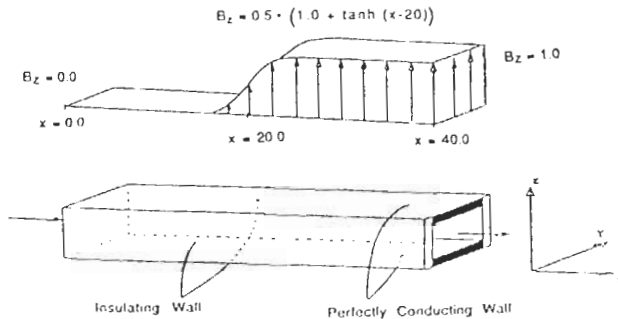


Fig. 6. Configuration of the applied magnetic field with the electric conductivities of the walls for the 3-D example problem. (Note that the method can handle arbitrary conductivities in all walls.)

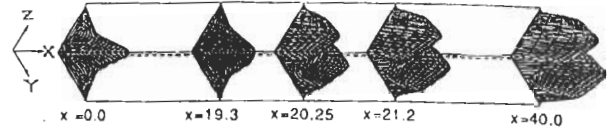


Fig. 7. Perspective view for axial velocity distributions.

this problem, two walls parallel to the magnetic field are electrically insulating and the others perpendicular to the field are perfectly conducting. Note that the method developed here can handle arbitrary electrical conductivities in all walls. The problem presented here is only an example. The nondimensionalized magnetic field is practically zero near the entrance and increases rapidly near the half span of the duct and reaches the uniform value of 1. The cross-section of the flow region has a nondimensionalized area $L_y \times L_z = 1 \times 1$ with a length of the duct $L_x = 40$.

The numerical work has been carried out for $N = 100$, $M = 100$, and $Re = 100$; a $23 \times 43 \times 43$ unequally spaced grid system is used in such a manner that fine grids are used near the walls and where the magnetic field changes rapidly.

At the entrance, the fully developed axial velocity profile obtained without a magnetic field is given, the transverse velocity components being zero. Also, the electrical potential is given to be zero at the entrance.

A perspective representation of the axial velocity distribution is seen in fig. 7, which shows the changes in the velocity profiles as the fluid flows from the entrance, with no magnetic field, to the exit with the uniform field. The features of a fully developed velocity profile can be seen in fig. 8. Here the M-shaped velocity profile in the plane perpendicular to the magnetic field

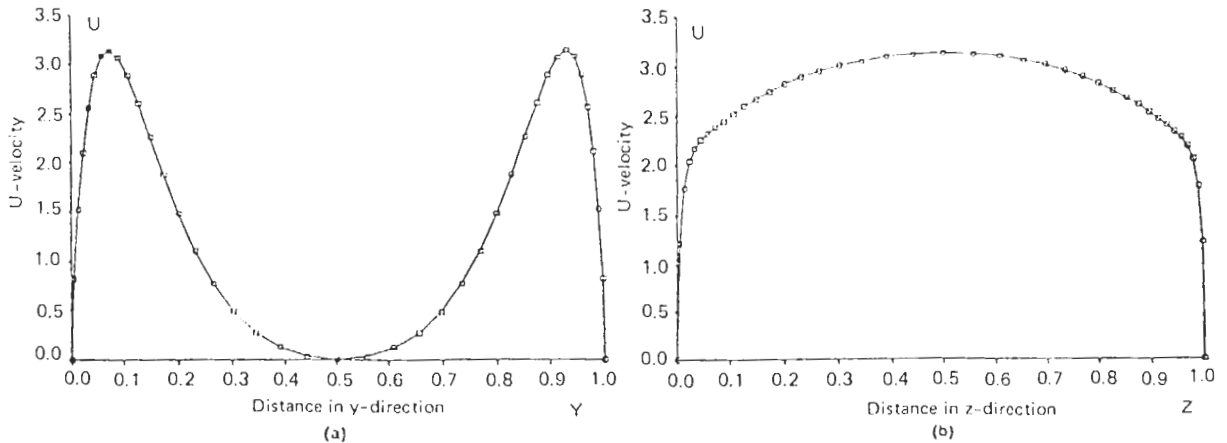


Fig. 8. Fully developed profiles for the axial velocity. (a) M-shaped profile in a plane perpendicular to the magnetic field, and (b) profile in the side layer in a plane perpendicular to the field.

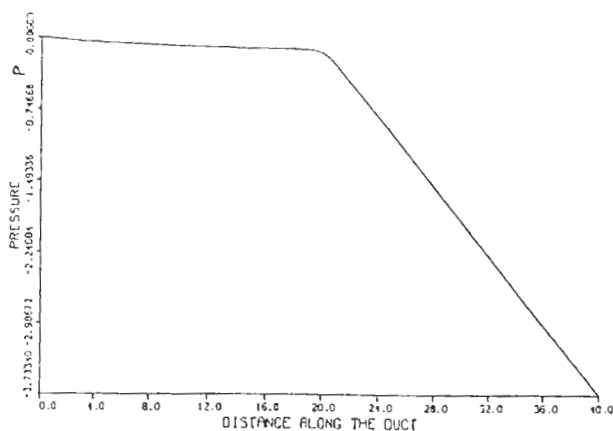


Fig. 9. Pressure distributions along the center of the duct.

and the axial velocities on the side-layer are shown. The marks on the curves in the figures denote the positions of the grid points for the velocity variable.

The pressure distributions along the center of the duct are shown in fig. 9. The pressure decreases slowly due to only the viscous friction in the region with the negligible magnetic field, while it drops rapidly due to the magnetic force term $J \times B$ where the magnetic field is not small.

4. Conclusion

A numerical method for the full solution of liquid metal MHD flows has been developed, and examples of two- and three-dimensional MHD flow problems using this numerical method have been presented. The complete governing equations, including the inertia and viscosity terms, have been solved by means of the finite difference method. An iterative technique has been used to determine the steady state solution.

The results indicate that M-shaped velocity profiles form when the fluid passes over a region of concentrated magnetic field in plane channel flows. Also, in rectangular duct flow, the velocity structure is changed near the nonuniform region of the magnetic field, becoming an M-shaped velocity distribution under the uniform field.

The two- and three-dimensional calculations presented in this paper are relevant for the design and operation of MHD devices including the liquid metal blanket in a fusion reactor which is expected to operate at high interaction parameters and Hartmann numbers.

The present numerical method, which used the con-

trol volume approach with the staggered grid system, is believed to have the capability of treating MHD flows with higher M and N with a reasonable speed of convergence. This capability will be tested later, together with unsteady, complex-geometry flows. Heat and mass transfer problems in MHD flows, where the full solution is particularly needed, will be pursued using a similar numerical method.

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Nomenclature

B_0	reference magnetic field intensity,
B	magnetic field intensity,
J	electric current,
e	tolerance,
L_0	reference length,
M	Hartmann number,
N	interaction parameter,
p	pressure,
Re	Reynolds number,
U_0	reference velocity,
U	velocity,
X	position,
ρ_0	reference density of the fluid,
σ	electric conductivity,
Φ	electric potential.

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