NUMERICAL METHOD FOR FLUID FLOW AND HEAT TRANSFER IN MAGNETOHYDRODYNAMIC FLOW

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ABSTRACT

A new numerical algorithm was developed to provide a fully detailed flow field in liquid metal MHD flow with a relatively large Hartmann number and interaction parameter. The algorithm includes the effects of advection and diffusion, and is capable of predicting momentum and heat transfer in MHD flows. Using this algorithm, an incompressible, viscous, three-dimensional MHD flow in a square duct is investigated at a low magnetic Reynolds number by means of the finite volume method. The velocity and temperature profiles are obtained in the developing region for constant wall temperature. The result shows that large velocities are obtained near the insulating walls parallel to the magnetic field. Also, near the perfectly conducting walls perpendicular to the field, a velocity profile like a Hartmann layer is obtained. In association with the velocity profiles, Nusselt number at the insulating walls (with side layer) is seen to be larger than that at the perfectly conducting walls (with Hartmann layer).

I. INTRODUCTION

Development of accurate and efficient methods for predicting MHD flow and heat transfer in a strong magnetic field is required for the design of MHD devices, including the liquid metal blanket in a fusion reactor. It has been shown[1] that there is a strong need for capabilities to predict MHD flow fields in fusion reactor blankets at relatively large values of interaction parameter, N, and Hartmann number, M.

In the application of MHD flow in fusion reactor blankets, lithium-containing liquid metal has good breeding and heat transfer characteristics. Two major issues are associated with liquid metal MHD flow in the presence of the magnetic field. One is pressure drop and the other is the capability of heat transfer.

Information on pressure drop and heat transfer is necessary for feasibility, economic, and safety reasons. With reduced pumping power for the pressure drop in liquid metal, the efficiency of the fusion reactor will be increased. The stress in the coolant duct will be reduced with decreased pressure drop, increasing the potential that a blanket design in which stresses are below the allowable can be developed. Predicting heat transfer characteristics requires knowledge of fluid flow. Heat transfer affects temperatures which are constrained by considerations of corrosion and radiation effect in materials.

Recently, some numerical methods[2] have been developed for MHD flow which neglect the viscous and inertial effects. However, when the magnetic field is changing in space or in time, and when the flow geometry is not simple, the viscous and inertial effects cannot always be neglected. For example, when the ratio of Hartmann number to interaction parameter is large, the inertial effect may not be negligible.[3]

Numerical solution of the full set of fluid and electromagnetic equations will give the most accurate solution for any situation. However, for large interaction parameters and large Hartmann numbers, the ratio of each of the convection and diffusion terms to the magnetic force is small. These different orders of magnitude of the terms in the equation of motion leads to difficulties in obtaining converged solutions. Because of these difficulties, few results[4,5] have been reported for computational MHD flow.

This work is concerned with the development of an efficient algorithm using the finite volume method for obtaining the full MHD solution of flow field and heat transfer for values of M and N and for geometries relevant to blankets in magnetic fusion reactors. In the following sections, non-dimensional governing equations, boundary conditions, and the numerical procedure are presented. These are followed by the numerical results for velocity, pressure, electric potential and heat transfer rates in a simple rectangular duct.

II. GOVERING EQUATIONS FOR LIQUID METAL MHD FLOW AND HEAT TRANSFER

In order to model convective heat transfer for MHD flows, Maxwell's equation and the equations of motion and energy must be considered. A steady-state, incompressible, constant-property, laminar flow of an electrically conducting fluid in the presence of a magnetic field is considered. The system of the equations governing the convective heat transfer can be written as:

Conservation of mass $$\nabla \cdot \vec{u} = 0 \qquad (1.a)$$

Equation of motion $$\rho \vec{u} \cdot \nabla \vec{u} = -\nabla p + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{u} \qquad (1.b)$$

Conservation of charge $$\nabla \cdot \vec{J} = 0 \qquad (1.c)$$

Ohm's law $$\vec{J} = \sigma[-\nabla \phi + \vec{u} \times \vec{B}] \qquad (1.d)$$

Induction equation $$0 = (\vec{B} \cdot \nabla)\vec{u} - (\vec{u} \cdot \nabla)\vec{B} - (\nabla \cdot \vec{u})\vec{B} + \alpha_m \nabla^2 \vec{B} \qquad (1.e)$$

Equation of energy $$\rho c \vec{u} \cdot \nabla T = k \nabla^2 T \qquad (1.f)$$

where α_m in the induction equation is defined by $\alpha_m = (\mu_o \sigma)^{-1}$, μ_o and σ being the magnetic permeability and electrical conductivity, respectively. In order to non-dimensionalize the system of equations, the variables are scaled as

$$\vec{x}^* = \frac{\vec{x}}{L_o} \quad (2.a)$$

$$\vec{u}^* = \frac{\vec{u}}{U_o} \quad (2.b)$$

$$\vec{B}^* = \frac{\vec{B}}{B_o} \quad (2.c)$$

$$\vec{J}^* = \frac{\vec{J}}{\sigma U_o B_o} \quad (2.d)$$

$$\phi^* = \frac{\phi}{U_o L_o B_o} \quad (2.e)$$

$$p^* = \frac{p}{\sigma U_o L_o B_o^2} \quad (2.f)$$

$$T^* = \frac{T - T_i}{T_w - T_i} \quad (2.g)$$

Using the above variables, given (2.a) to (2.g), MHD flows can be well-scaled when the pressure force term is larger than the inertial and viscous terms. If the magnetic force terms are as large as, or smaller than the inertial and viscous terms, the pressure can be scaled based on the dynamic pressure ρU_o^2 instead of $\sigma U_o L_o B_o^2$. Also, the temperature is scaled based on the temperature difference at the walls and at the inlet flow (both are specified).

From the scaling of each variable, a system of non-dimensional governing equations is derived together with the dimensionless parameters which govern the flow characteristic of MHD flows. The non-dimensionalized version of the system of equations can be written as

Conservation of mass $\nabla^* \cdot \vec{u}^* = 0$ (3.a)

Equation of motion
$$(\vec{u}^* \cdot \nabla^*)\vec{u}^* = N(-\nabla^* p^* + \vec{J}^* x \vec{B}^*) + \frac{1}{Re}\nabla^{*2}\vec{u}^* \quad (3.b)$$

Conservation of charge $\nabla^* \cdot \vec{J}^* = 0$ (3.c)

Ohm's law $\vec{J}^* = -\nabla^* \phi^* + \vec{u}^* x \vec{B}^*$ (3.d)

Induction equation
$$0 = (\vec{B}^* \cdot \nabla^*)\vec{u}^* - (\vec{u}^* \cdot \nabla^*)\vec{B}^* - (\nabla^* \cdot \vec{u}^*)\vec{B}^* + \frac{1}{Re_m}\nabla^{*2}\vec{B}^* \quad (3.e)$$

Equation of energy $(\vec{u}^* \cdot \nabla^*)\vec{T}^* = \frac{1}{RePr}\nabla^{*2} T^*$ (3.f)

where the dimensionless parameters N, Re, Re_m and Pr are the interaction parameter, Reynolds number, magnetic Reynolds number, and Prandtl number respectively. They can be written as

$$Re = \frac{U_o L_o}{\nu} = \frac{(Inertial\ force)}{(Viscous\ force)} \quad (4.a)$$

$$N = \frac{\sigma B_o^2 L_o}{\rho_o U_o} = \frac{(Magnetic\ force)}{(Inertial\ force)} \quad (4.b)$$

$$Re_m = \mu_o \sigma U_o L_o = \frac{U_o L_o}{\alpha_m} = \frac{(Induced\ magnetic\ field)}{(Applied\ magnetic\ field)} \quad (4.c)$$

$$Pr = \frac{\nu}{\alpha} = \frac{(diffusion\ rate\ of\ kinematic\ viscosity)}{(diffusion\ rate\ of\ thermal\ diffusivity)} \quad (4.d)$$

The Hartmann number, M, can be defined as a square root of the product of Re and N as

$$M^2 = (Re \cdot N) = \frac{\sigma B_o^2 L_o}{\mu} = \frac{(Magnetic\ force)}{(Viscous\ force)} \quad (4.e)$$

In most practical liquid metal MHD, the magnetic Reynolds number is very small, which means that the induced magnetic field is negligible compared with the applied field. Therefore, the induction equation shown in Eq. (3.e), which governs the induced magnetic field, will not be used any longer.

In order to use the numericalmethod to be presented in the next section, the system of equations can be modified. (From this point, the asterisk which means that the quantity is non-dimensionalized will be deleted for convenience.)

Taking the divergence of Eq. (3b) and substituting Eq. (3.a) yields a pressure equation of the Poisson-type:

$$\nabla^2 p = \nabla(\vec{J} x \vec{B}) + \frac{1}{N}\nabla \cdot \left((\vec{u} \cdot \nabla)\vec{u}\right) \quad (5)$$

where the pressure has an elliptic behavior with source terms of the magnetic forces and velocity gradients.

A similar equation for electric potential can be derived by substituting Eq. (3.d) into Eq. (3.c)

$$\nabla^2 \phi = \nabla(\vec{u} x \vec{B}) \quad (6)$$

which also has an elliptic behavior of the electric potential with a source term of the electromotive forces.

From these manipulations of the governing equations, a new system of equations, which will be used in the numerical procedure, can be given as:

Equation of motion $(\vec{u} \cdot \nabla)\vec{u} = N(-\nabla p + \vec{J} \times \vec{B}) + \frac{1}{Re}\nabla^2 \vec{u}$ (7.a)

Equation for pressure $\nabla^2 p = \nabla \cdot (\vec{J} \times \vec{B}) + \frac{1}{N}\nabla \cdot \left((\vec{u} \cdot \nabla)\vec{u}\right)$ (7.b)

Equation for potential $\nabla^2 \phi = \nabla \cdot (\vec{u} \times \vec{B})$ (7.c)

Conservation of mass $\nabla \cdot \vec{u} = 0$ (7.d)

Conservation of charge $\nabla \cdot \vec{J} = 0$ (7.e)

Ohm's law $\vec{J} = (-\nabla \phi + \vec{u} \times \vec{B})$ (7.f)

Equation of energy $(\vec{u} \cdot \nabla)T = \frac{1}{RePr}\nabla^2 T$ (7.g)

The first three equations will be solved for velocity, pressure and electric potential, respectively. The following three equations will be used as corrective steps for the implementation of the principles of a global conservation along the main direction of flow and of a local conservation of each computational cell.

III. BOUNDARY CONDITIONS

For the velocity field, no-slip and no-penetration conditions are given at the interface between the fluid region and the duct walls as

$$\vec{u} = 0 \quad (8)$$

The boundary condition for the pressure, which comes from the equation of motion, can be written as:

$$\frac{\partial p}{\partial n} = (\vec{J} \times \vec{B})_n + \frac{1}{M^2}\frac{\partial^2}{\partial n^2}(\vec{u})_n \quad (9)$$

The numerical code is written so as to handle the conjugate problem where the electric currents and temperature field are calculated together in the regions of fluid and duct walls. The boundary condition for the electric potential is such that the whole system including the regions of fluid and duct walls are electrically insulated from the outside, which can be written, at the outer surface of the duct walls, as

$$\frac{\partial \phi}{\partial n} = 0, \text{ at the outer surface} \quad (10)$$

The constant wall temperature is specified at the outer surface of the duct walls. Therefore, from normalization of the temperature field, the boundary condition can be written as:

$$T_w = 1, \text{ at the outer surface} \quad (11).$$

Governing equations (7.a) to (7.f), along with the boundary conditions given in Eqs. (8) to (10) constitute a system of coupled, non-linear partial differential equations for the dependent variables $\vec{u}^*$, $\vec{J}^*$, $\vec{p}^*$, $\vec{\phi}^*$. Energy equation (7.g) with the boundary condition given in Eq. (11) can be solved after the velocity field is obtained.

IV. NUMERICAL METHOD

Some flow conditions can be considered where a large body force, which is not expressed in terms of the velocity and pressure, is involved. If this force includes a variable (which will be called third variable in the coupling) satisfying a physical law in the flow field, the coupling should account for the effect of this body force. The examples of such flow conditions will include liquid metal MHD flows with large interaction parameters and natural convection flows with large Grashof numbers. In the liquid metal MHD flows, the third variable could be either the electrical current density or the electrical potential depending on the formulation of the governing equation. And, in the natural convection flows, a difference in temperature in density would be the third variable.

In order to avoid the possible numerical divergence in the pressure-velocity coupling when the magnitude of the third variable in the momentum equation is larger, a pressure equation of Poisson type, which is obtained by taking the divergence of the equation of motion and by substituting the equation of mass conservation, will be treated directly. In this method, the pressure field is calculated taking into account all the effects of inertial, viscous and the other body forces with their own magnitudes.

A new solution method including the idea of the coupling of velocity, pressure and the third variable is developed to properly treat a large body force which may be involved in the flow field.

The features of the method include:

1) fully implicit procedure;
2) the use of primary variables;
3) the control volume approach with the staggered grid system;
4) the implementation of the discretization of the differential form of the Poisson-type pressure equation derived from the divergence of the equation of motion and substituting the continuity equations;
5) the splitting of the operations of equations to decouple the link among pressure, velocity and the third variable, so that only one variable is calculated at a time;
6) the use of the hybrid method for the treatment of the convective and diffusive fluxes; and
7) as a corrective step, the implementation of the principles of a global conservation along the main direction of flow and of a local conservation on each computational cell.

This method is not believed to be the most economical procedure for the simulation of the ordinary fluid flow. However, the method would be very effective for the flow field where the body force, being a variable, is at least as important as the inertial, viscous and pressure forces.

V. RESULTS AND DISCUSSION

A three dimensional MHD flow in a straight duct with a rectangular cross-section is considered with an applied magnetic field in the z-direction as shown in Figure 1. The method can handle a completely general field. However, for the purpose of demonstration a simple single-component field is used.

For this problem, two walls parallel to the magnetic field are electrically insulating and the others perpendicular to the field are perfectly conducting. Note that the method developed here can handle arbitrary combinations of the electric and thermal conductivities in all walls and fluid. The problem presented here is only an example. The non-dimensionalized magnetic field is practically zero near the entrance and increases rapidly near the half span of the duct and reaches unity.

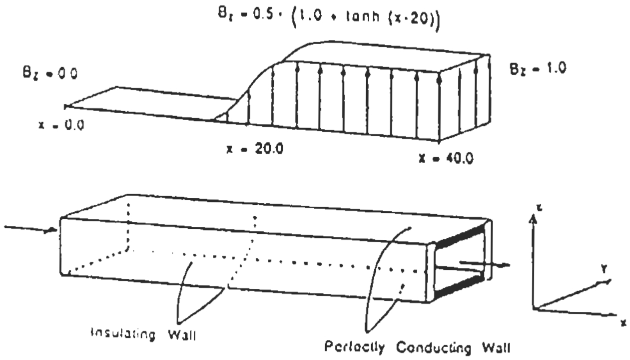

Fig. 1. Configuration of the applied magnetic field with the electric conductivities of the walls for the 3-D example problem (Note the method can handle arbitrary conductivities in all walls)

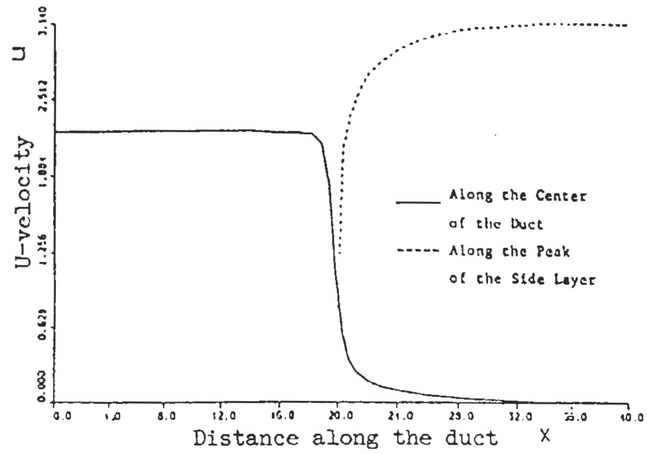

Fig. 3 Axial velocity distribution along the center of the duct and along the side layer

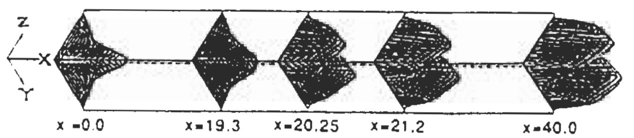

Fig. 2 Perspective view for axial velocity distributions

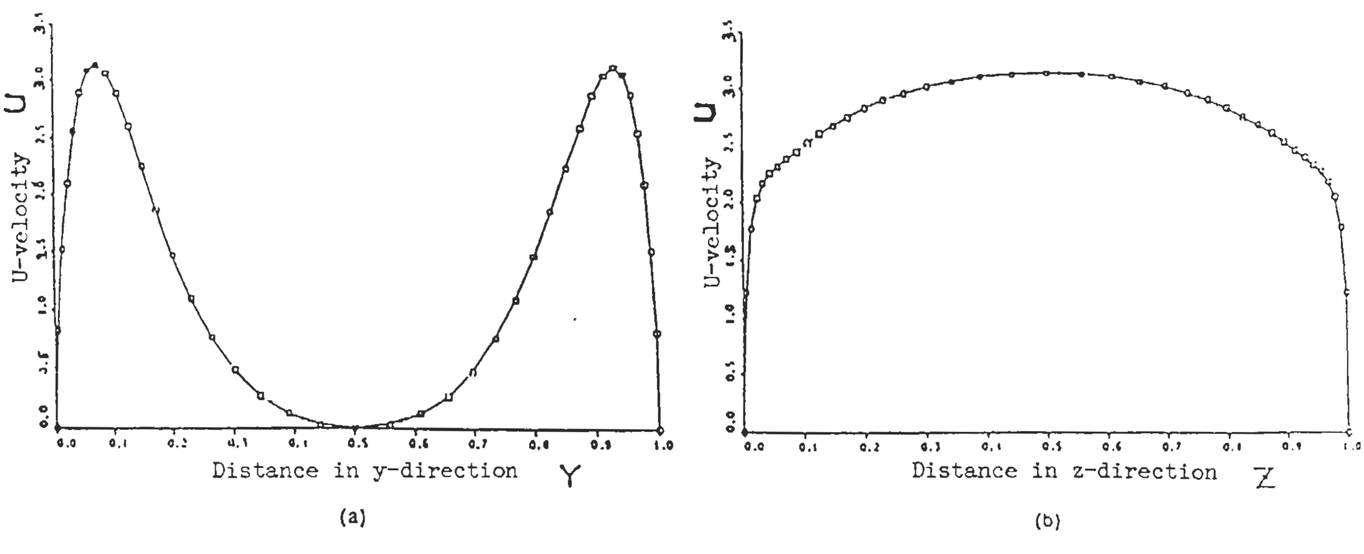

Fig. 4 Fully developed profiles for axial velocity: a) M-shaped profile in plane perpendicular to the magnetic field, and b) profile in the side layer in plane perpendicular to the field

At the entrance some appropriate boundary conditions are given. Fully developed axial velocity profile obtained without magnetic field is given, the transverse velocity components being zero. Also the electrical potential is given to be zero at the entrance. The non-dimensionalized inlet temperature ($T_i = 0.0$) is specified while the constant wall temperature ($T_w = 1.0$) is given at the outer surface of the duct walls. The conductivity of the duct walls are assumed to be the same as that of the fluid.

The cross section of the flow region has a non-dimensionalized area of $L_y \times L_z = 1 \times 1$ with a length of the duct $L_x = 40$. The numerical work has been carried out for N =100, M = 100, Re = 100 and Pe = 150. In order that the temperature field is not dominated entirely by the conduction nor by the convection, some arbitrary value fo Peclet number is picked up for the energy equation by considering the value of Reynolds number and the geometrical size of the problem domain. A 23 x 43 x 43 unequally spaced grid is used in such a manner that the fine grids are used near the walls and where the magnetic field changes rapidly.

The computational domains for the pressure and the velocities are confined to fluid regions. As for the electric potential, the conjugate problem is considered so that the potential equation is solved simultaneously in the regions of fluid and walls. With this formulation, the boundary condition for the potential at the interface between the fluid and the wall will not be used. Instead, the condition used for the electric potential is that the whole system, including the fluid and the walls, is insulated from the outside.

For the temperature field, the conjugate-problem approach is possible for the constant wall temperature boundary condition given at the outer surface of the duct wall. For the constant heat flux boundary condition, the conjugate-problem approach may not be used.

Figure 2 shows the perspective representation of the axial velocity distribution where the changes in the velocity profiles are seen as the fluid flows from the entrance with no magnetic field to the exit with the uniform field. The axial velocity distributions along the center of the duct and along the peak of the side layer are shown in Figure 3, where the axial velocity along the center of the duct decreased to almost zero at the exit, while the peak axial velocity on the side layer increases to more than three times the mean velocity at the same location. The feature of the fully developed velocity profile under the magnetic field can be seen in Figure 4. Here the M-shaped velocity profile in the plane perpendicular to the magnetic field and the axial velocities on the side-layer are shown. The marks on the curves in the figures denote the positions of the grid points for the velocity variable.

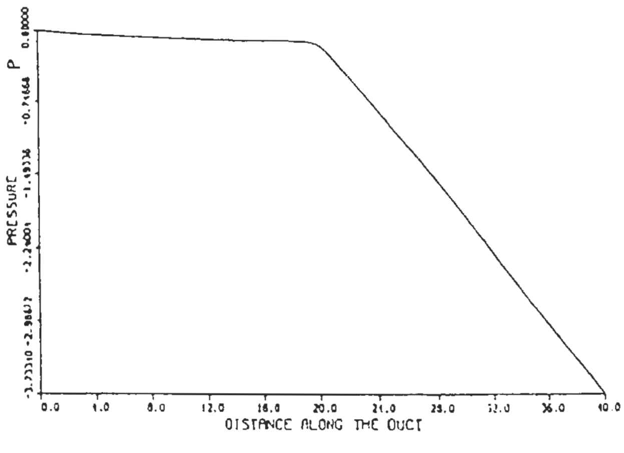

Fig. 5 Pressure distributions along the center of the duct

Figure 5 shows the pressure distributions along the center of the duct. The pressure decrease is not steep in the region with the negligible magnetic field due to only the viscous friction, while it becomes rapid where the magnetic field is not small due to the magnetic force term J x B.

On the insulating walls where the magnetic field is not small, large electric potentials are induced positively and negatively respectively, or two insulating walls facing each other. Figure 6 shows these positively induced potentials along the center of the insulating wall.

Where the applied magnetic field is not small, side layers with fast velocity form near the insulating walls, as shown in Figure 4. It is well known that fast velocities near walls increase the heat transfer rate.

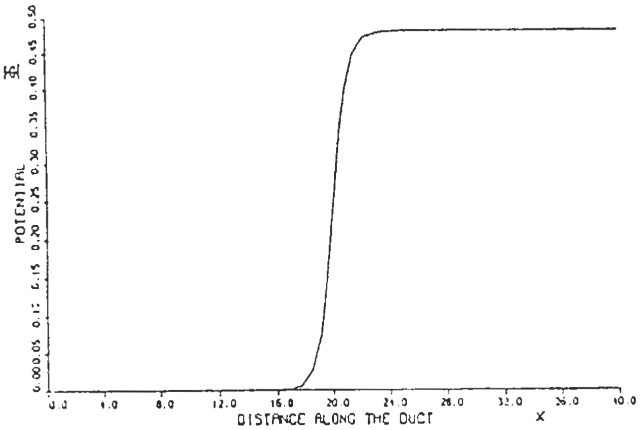

Fig. 6 Electric potential distribution along the centerline of the insulating walls

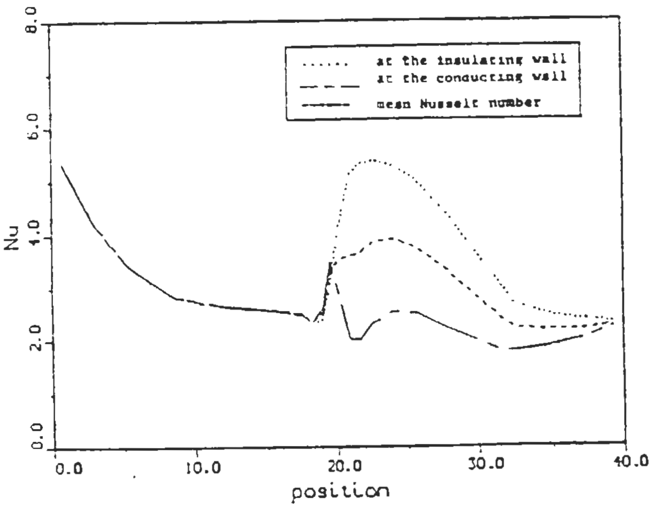

Fig. 7 Variation of local Nusselt number in a rectangular duct with constant surface temperature

The effect of the side layer (with fast velocity) on the heat transfer rate is shown in Figure 7, where the Nusselt numbers at the insulating and perfectly conducting walls and mean Nusselt number are indicated. Non-dimensionalized heat transfer rate at the insulating wall (with side layer) is seen to be much larger than that at the Hartmann layer as the fluid enters into the regions of the magnetic field. Later, the heat transfer rates at the side layer and at the Hartmann layer become close to each other as the bulk fluid temperature, T_{bulk}, approaches the specified wall temperature, T_{wall} (=1.0), where the convection effects cannot play an important role in the heat transfer.

VI. CONCLUSION

A numerical method for the full solution in liquid metal MHD flows has been developed, and using this method, characteristics of the fluid flow and heat transfer have been presented for a three-dimensional MHD flow in a rectangular duct for constant wall temperature.

The results indicate that the velocity structure is substantially changed as the fluid passes through the non-uniform regions of the magnetic field, and forms side layers on the electrically insulating walls. Nusselt number on the walls with side layer is seen to be higher than that on the walls with Hartmann layer as long as the convective heat transfer effects are important.

The test problem shown here was for a combination of a simple geometry and a single-component magnetic field. MHD flow in more complex geometries and/or under complex magnetic fields can be treated using the same algorithm presented in this paper. Examination of these complicated MHD flows is continuing.

The present numerical method is believed to have the capability of treating MHD flows with higher interaction parameter and Hartmann number with a reasonable speed of convergence. This capability will be tested later, together with mass transfer problems in MHD flows where the full solution is particularly needed.

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