

Novel liquid blanket configurations and their hydrodynamic analyses for innovative confinement concepts

K. Gulec ^{a,*}, M.A. Abdou ^a, R.W. Moir ^b, N.B. Morley ^a, A. Ying ^a

^a *Mechanical and Aerospace Engineering Department, UCLA, Los Angeles, CA 90095, USA*

^b *Lawrence Livermore National Laboratory, Livermore, CA, USA*

Abstract

Hydrodynamics analyses as a part of the APEX (advanced power extraction) study demonstrates the potential application of applying swirling thick liquid walls to innovative confinement concepts such as field reversed configuration (FRC), spherical torus (ST) and heavy-ion fusion (HIF). This paper addresses the design and the hydrodynamic aspects of fusion relevant swirling flow including 3-D velocity distribution, variations of the flow height in axial and azimuthal directions and hydrodynamic flow stability. Numerical hydrodynamic analyses using a 3-D code with Flibe as the working fluid, shows that a thick liquid first-wall/blanket (> 0.6 m) can be maintained in a circular vacuum chamber of 2 m radius by injecting the liquid layer from one side through a swirl flow generating inlet with axial (7 m/s) and azimuthal (10 m/s) velocity components. Parametric computational study indicated that the liquid layer thickness in axial and azimuthal directions is strongly dependent on the inlet axial, azimuthal velocity values and gravitational acceleration. It also shows that a uniform liquid layer thickness can be maintained for axial and azimuthal inlet velocities of 11 and 13 m/s in a cylindrical chamber with a 2 m radius and 12 m length. The swirling liquid wall idea is applied successfully to ST and HIF configurations. A 2D linear stability analysis using potential flow theory (Reynolds number is $\sim 10^6$ for liquid wall thickness of ~ 0.5 m) of the swirling flow in the azimuthal flow direction suggested that mean flow is stable when the surface tension, gravitational acceleration, and the centrifugal force effects are considered. © 2000 Published by Elsevier Science B.V.

Keywords: Azimuthal; APEX; Heavy-ion diffusion

1. Introduction

The present thick liquid FW/blanket idea for field reversed configuration (FRC) and spherical torus (ST) configurations evolved from the thick liquid wall (TLW) concepts proposed by Moir

[1,2] for magnetic fusion energy configurations. The idea is now being investigated, modified and developed in the APEX (advanced power extraction) study [3], which is aimed at exploring innovative concepts of handling high power density that may enhance the potential for fusion to be a competitive energy source. A neutronically thick liquid wall (~ 0.5 m when Flibe is used as an operating fluid) may have advantages: (1) high fluence capability (no material thermal stress); (2)

* Corresponding author. Tel.: +1-310-2062560; fax: +1-310-8252599.

E-mail address: kgulec@fusion.ucla.edu (K. Gulec).

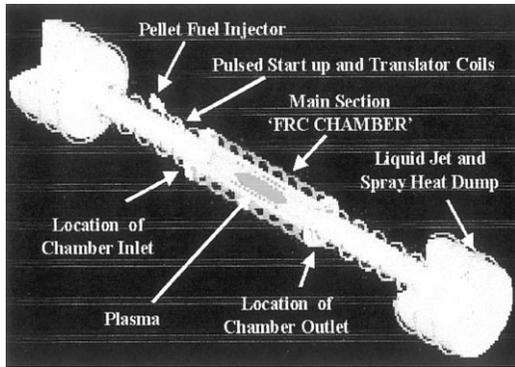


Fig. 1. General layout of preliminary FRC reactor design.

lower radiation damage and reduced activation; (3) simplified maintenance and a lower failure rate. In the present study, the swirling liquid layer idea is explained both analytically and computationally in detail for the FRC configuration, and its applications to the ST and heavy-ion fusion (HIF) configurations are also presented. The conceptual FRC configuration has more than a 25 m length, which includes the main section (where plasma exists) located at the center, the two liquid jets (i.e. spray heat dump) and pulsed start-up

sections located on both sides of the main section (Fig. 1). A thick liquid first-wall/blanket (> 0.5 m) can be established and maintained in the main section by injecting the liquid layer at one side of the circular vacuum chamber through a swirl flow generating inlet with axial and azimuthal velocity components (Fig. 2). Then, the liquid layer adheres to the inner walls of the cylindrical chamber by means of centrifugal acceleration ($> \sim 3.2 \times g$), which results from its high azimuthal velocity (> 8 m/s) and the small radius of curvature of the cylindrical chamber (< 2 m). As an example, the inner wall of the main section (with radius of 2 m) can be covered by a fast moving liquid layer of ~ 0.5 m thickness with azimuthal and axial velocities of 8–12 m/s azimuthal and at > 8 m/s respectively. The liquid layer is diverted towards the outlet by centrifugal acceleration on the diverging conical back wall structure that is located at the other end of the main section. Swirl flow makes approximately one rotation during its travel between the inlet and outlet sections.

It should be noted that the idea presented here is in the exploratory phase and may be developed into a mature design concept when the several fundamental issues stated in this paper are ad-

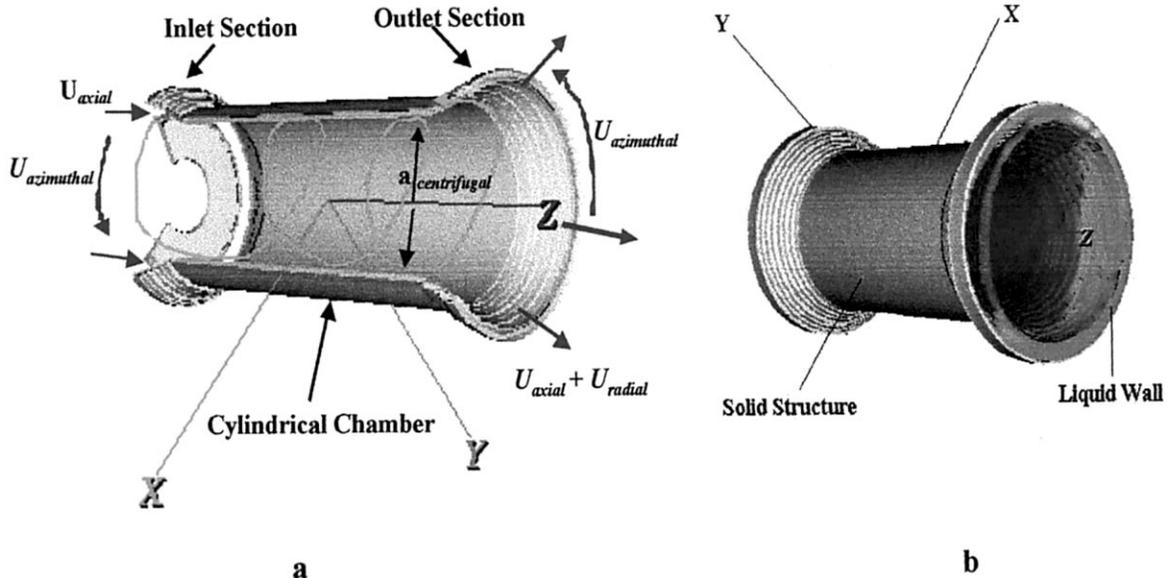


Fig. 2. (a) Illustration of swirl flow mechanism in the main section with converging inlet and diverging outlet sections. (b) The 3-D fluid distribution of FRC swirl flow. (result of CFD simulation.)

dressed in the future phase of the current study. In the preliminary analysis the constraints are: (1) liquid blanket thickness (> 0.45 m); (2) velocity at the liquid wall surface facing to the plasma (> 8 m/s); (3) elimination of liquid splash or drip to the plasma. The design variables for preliminary study are: (1) axial and azimuthal inlet velocities; (2) chamber radius; and (3) lengths and slopes of inlet/outlet converging/diverging sections.

In the present study, ongoing studies about the preliminary designs of the inlet nozzle, outlet recovery section and related hydrodynamic characteristics are not presented.

2. Hydrodynamic analyses and simulation

The primary condition for free vortex flow in a cylindrical chamber is that the centrifugal force pushing the fluid towards the wall should be greater than the weight of the fluid.

$$\int_{R_s}^{R_b} \rho \frac{U_{\theta}^2(r)}{r} r dr d\theta > \int_{R_s}^{R_b} \rho \cdot \vec{g} \cdot r dr d\theta \quad (1)$$

where R_b is the radius of the cylindrical chamber, R_s is the radius of the flow surface, g is the gravitational acceleration, ρ is the operating fluid density and $d\theta$ is the unit angle in the neighborhood of 90° (where gravitational acceleration direction is in the opposite direction of centrifugal acceleration). The minimum required azimuthal surface velocity for the liquid layer to swirl inside the chamber is:

$$C^2 > g \frac{[R_b + R_s] R_b}{2 R_s} \quad (2)$$

where C is the azimuthal velocity at the flow surface when the velocity profile of potential flow in the azimuthal direction is assumed as $U_{\theta} = C \cdot R_s/r$ [4,5]. The minimum required azimuthal surface velocity is 4.8 m/s for a chamber radius of 2 m and liquid wall thickness of 0.5 m. However, computational results showed that there is a 35% difference in the liquid layer thickness between the flow at the bottom (270°) and top (90°) sections. This non-uniformity in the liquid layer thickness on planes perpendicular to the flow direction is due to gravitational acceleration and can be mini-

mized using the relationship $U_{\theta}^2/(2gR_s) + 0.5 > 10$. This relationship is determined to satisfy the condition of $\Delta\vec{a}/\vec{a} < 0.1$ where \vec{a} is the total acceleration, including the gravitational acceleration, effecting the flow in the radial direction and $\Delta\vec{a}$ is the difference in the radial acceleration between the top and bottom sections. Analytical derivations suggest that the effect of gravitational acceleration on the hydrodynamic characteristics of the liquid jet can be minimized to $U_{\theta} > 13.6$ m/s assuming a liquid layer thickness of a 0.5 m and chamber radius of 2 m.

Numerical hydrodynamic analyses are performed using Flow3d code [6]: (a 3-D time dependent Reynolds Averaged Navier Stokes (RANS) solver with a volume of fluid (VOF) free surface tracking algorithm). An electrically low conducting ($150 \text{ } 1/\mu\text{m}$) molten salt: Flibe (34% Be_2F 66% LiF) is used as the operating fluid. Preliminary design exploration study is performed using constant axial azimuthal and axial inlet velocities to a converging (radius converges from a 2.75-m radius to 2 m within a 1.5-m axial flow length) chamber that opens into a constant radius cylindrical chamber.

As seen in Figs. 3 and 4, base operating variables and dimensions for the FRC configuration of 10 m/s axial and 10 m/s in azimuthal inlet velocities can maintain a certain liquid layer thickness along the flow axis. Results also indicate that there exists a continuous decrement in the liquid layer thickness from 0.85 to 0.6 m in the axial direction. This may be due to the free outlet boundary condition at the exit of the main section. The decrease in the flow thickness in the axial direction may be minimized by increasing the axial inlet velocity or using a slight contraction in the cylindrical chamber radius towards the outlet that moves the cyclotropic pressure balance towards the axis of the chamber. The velocity distribution in the axial direction confirms that both convergent inlet and divergent outlet sections may work properly. However, better design is needed for the convergent section. As the chamber radius decreases, the centrifugal acceleration effect on the flow increases. This condition results in a thicker liquid layer. Although this hydrodynamic shape close to the inlet may be desired for

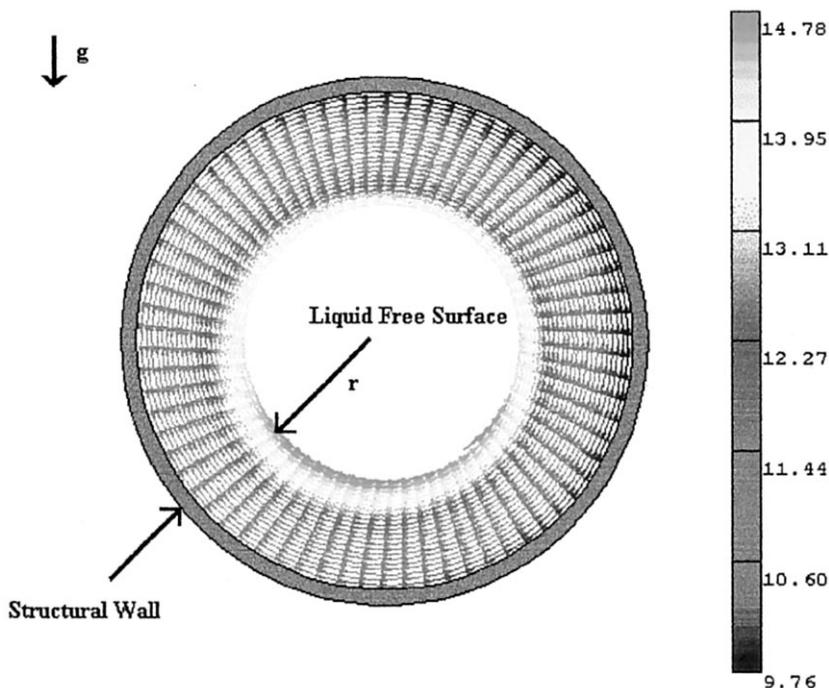


Fig. 3. 2-D (r) Velocity distribution and liquid layer height distribution in the azimuthal direction at an arbitrary axial location.

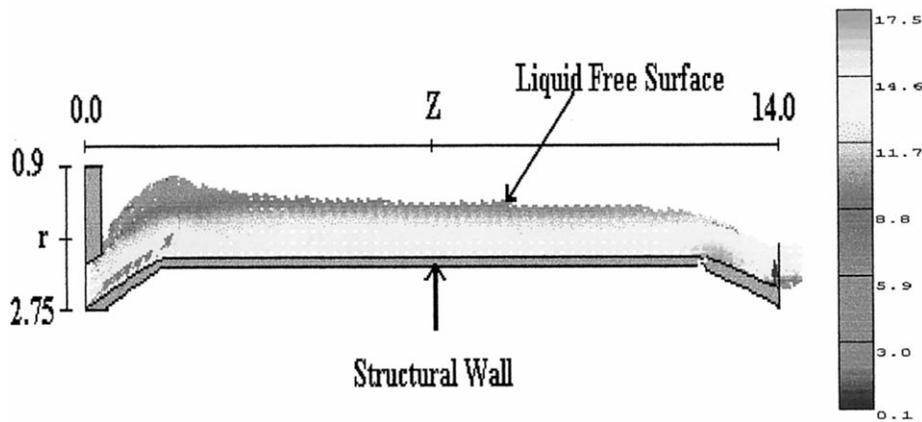


Fig. 4. 2-D (r - z) Velocity distribution and liquid layer height distribution in the axial direction at an arbitrary azimuthal location.

elimination of neutron streaming, thickening of the liquid layer results in lower axial surface velocity. This condition may not be desirable since the radiative heating becomes higher (increase in the liquid evaporation rate) the duration of the flow surface exposed to plasma increases.

Preliminary parametric computational analysis

for swirling/non-swirling with horizontally/vertically located FRC chamber configurations suggests that a horizontally located chamber with a swirl flow may be an attractive configuration. As seen in Tables 1–3, a horizontally located chamber configuration with swirl flow has advantages over a vertically located chamber configuration

because, it: (1) minimizes the required pumping power for high thermal efficiency; (2) can maintain a uniform blanket thickness in the chamber with no minimum/maximum velocity requirement in the horizontal direction; (3) may enable the use of a smaller chamber and extraction of higher power density when the heat transfer/heat transfer-enhancement at the flow free surface is addressed.

In addition, the advantages of swirl flow may be: (1) the achievement of higher thermal efficiency, for an inherent stratification in the flow may form due to the high centrifugal acceleration and may potentially ease separation of the hot

fluid (from the surface) from the colder one at the outlet section; (2) no structural requirement in the chamber for flow regulation.

3. Application of the swirl concept to other MFE/IFE configurations

The swirl concept may be applied to several configurations such as the ST for MFE and HIF [7] (Heavy Ion Facility) for IFE applications. The application of a swirling liquid layer to these configurations is explained in the following sections.

Table 1
Structural positioning of FRC chamber and varied operational parameters for parametric study

Relative positioning	Structural modification	Inlet radius (m)	Outlet radius (m)	Varied parameters
Horizontal	Yes	2.0	1.75	U_{axial} , $U_{\text{azimuthal}}$
Horizontal	No	2.0	2.0	U_{axial} , $U_{\text{azimuthal}}$
Vertical	Yes	2.0	1.75	U_{axial}
Vertical	No	2.0	2.0	U_{axial}

Table 2
Required operation conditions and observed blanket symmetry for horizontally and vertically located FRC chamber cases

Modification of FRC wall	Blanket symmetry	Rotational flow	Azimuthal velocity (m/s)	Axial velocity (m/s)	Velocity magnitude (m/s)
Horizontally positioned FRC	Yes	Required	> 11	> 10	> 14.8
Horizontally positioned FRC	No	Required	> 7.5; < 11	< 10	< 12.5
Vertically positioned FRC	Yes	No	–	> 20	> 20

Table 3
Required operation conditions and observed blanket symmetry conditions for horizontally and vertically located FRC chamber cases when the chamber structural wall topology is modified

Modification of FRC wall	Blanket symmetry	Rotational flow	Azimuthal velocity (m/s)	Axial velocity (m/s)	Velocity magnitude (m/s)
Horizontally positioned FRC	Yes	Required	> 8	> 8	> 11.4
Vertically positioned FRC	Yes	No	–	> 15	> 15

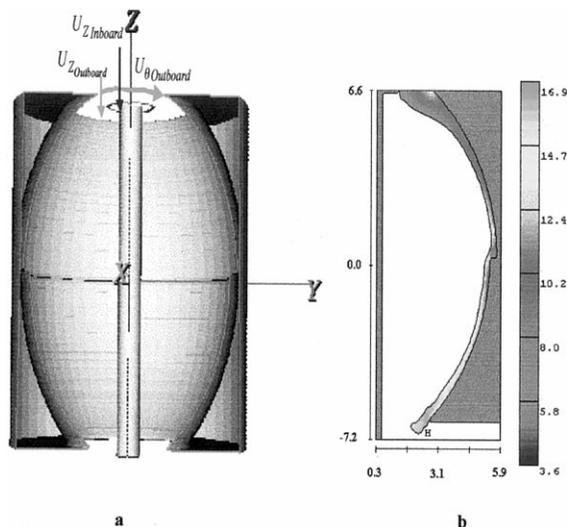


Fig. 5. (a) The structural modeling of ST geometry including the modification in the outboard topology. (b) 2-D velocity magnitude contour at r - z plane at the outboard and liquid layer height distribution in the z -direction at an arbitrary azimuthal angle.

3.1. Application of swirl concept to ST configuration

As seen in Fig. 5 (a), in the ST configuration, a thick liquid layer flows from the top of the reactor toward the outboard with vertical and azimuthal velocity profiles. The liquid layer becomes attached to the outer board due to centrifugal acceleration from the toroidal liquid layer velocity (Fig. 5 (b)). The formation of 0.2 m high step in the reactor mid-plane (where the effect of gravitational acceleration on the liquid layer thickness is the highest) on outboard vacuum vessel topology helps to keep the liquid layer thickness constant (> 0.3 m). The centrifugal acceleration (> 35 m/s²) pushes the fluid towards the outboard and prevents the flow deflection into the plasma region.

Preliminary results were obtained for the operating condition of $V_{\text{poloidal}} = 4.5$ m/s, $V_{\text{toroidal,ave}} = 12$ m/s from a 1.5 m annular with an inner radius of 1.8 m. This inlet condition was parametrically obtained by: (1) taking into account the substantial increase in the total flow area ($\sim 266\%$) in the vertical direction from the top of the reactor to

the mid-plane to obtain a 0.3 m minimum liquid layer thickness at the mid plane; (2) minimizing the poloidal velocity at the ST mid plane so that deflection of the liquid towards plasma is prevented. Using a thicker inlet fluid layer may compensate for the increase in the poloidal velocity due to gravitational acceleration (thinning of the liquid layer). Also, a thicker fluid layer with lower velocity requires less pumping power, as the pumping power is proportional to the velocity squared.

In the inner board, a fast annular liquid layer of 0.75 m thick (over an inner radius of 1 m) with 15 m/s vertical velocity is used. High vertical velocity ($V_z > 15$ m/s) prevents excessive thinning in the liquid layer ($< 30\%$).

The swirling liquid layer idea is more applicable to an ST configuration than an ARIES-RS configuration because the ST geometry has higher radius of curvature in the poloidal direction (~ 8.0 versus ~ 4.0 m) and has smaller radius of curvature in the toroidal direction than its' poloidal direction (5.0 as compared to 8.0 m). Toroidal rotation of the liquid layer may result in a substantial increase in the centrifugal acceleration of the flow towards back wall (266% increase at the inlet and 160% at mid plane). The ST is taller than the ARIES-RS (~ 12.0 versus ~ 6.0 m) so the effect of gravity in the hydrodynamic characteristics of the flow in the ARIES is more important and requires additional passive mechanism to overcome thinning.

3.2. Application of the swirl concept to heavy ion facility configuration

As seen in Fig. 6 (a), a cusp-like liquid surface may be formed by driving a swirling liquid layer into the cylindrical structural shell from both sides using an azimuthally symmetric outlet in the magnetic cusp region, so that the liquid streams are ejected by their own centrifugal momentum. The radius of the first liquid surface may be chosen to be large enough such that liquid fracture due to neutron isochoric heating may be avoided, and the thickness of the liquid flow is chosen such that it reduces the damage to structural materials to an acceptable level. The reduc-

tion of vaporized and spalled wall material allows for faster chamber clearing and an increased repetition rate.

A preliminary HIF configuration for hydrodynamic simulation analysis (Fig. 6 (b)) consists of two identical structures located 1 m away from each other, each having a 1.5 m long converging section (2.75 m radius to 2 m radius) on the far end, a 3 m long cylindrical section with 2 m radius and 1.5 m long diverging section (2 m radius to 2.75 m radius) towards the middle. Initial results are obtained for inlet velocities of $V_{\text{azimuthal}} = 12$ m/s, $V_{\text{axial}} = 2$ m/s. Preliminary analysis suggests that the swirling liquid wall in the cylinder and the cusp shaped liquid wall at the outlet sections can be maintained, and the formation of the cusp is strongly dependent on the azimuthal velocity component. The distance between the two cylindrical sections becomes lower and the fluid drip into the target section may be eliminated when the initial axial inlet velocity is minimized.

As seen in Fig. 7, there is an asymmetry in the cusp due to the effect of gravitational acceleration on the hydrodynamic behavior of the swirling liquid flow. The slope of the cusp is constant due to the structural constraint, however the thickness of the liquid wall on the diverging section varies due to the gravitational acceleration. Increasing the azimuthal flow velocity may minimize this

condition. The outlet velocity varies as the direction of gravitational acceleration changes with respect to the axial flow direction changes.

An optimization analysis for the diverging cusp section (angle, length) and liquid wall operating conditions will be performed to eliminate droplet formation, maximize heat transfer and maintain the wall thickness along the axial and azimuthal flow directions.

4. Mechanisms effecting the stability of swirl flow and perturbation sources

There are studies performed for high velocity flow over concave surfaces and swirl chambers (which is totally filled with fluid for combustion studies) separately [8–11]. The experimental information obtained from these studies suggests that the radial distribution of the tangential flow is divided into two regions, i.e. a region of forced rotational flow in the center of the chamber surrounded by a region of quasi-free rotational flow. The proposed FRC swirling liquid layer configuration does not have a forced vortex region close to the axis of the chamber. The liquid layer flow in the azimuthal and axial directions have high Reynolds numbers ($Re > 10^6$) where the boundary layer thickness becomes comparably less than the

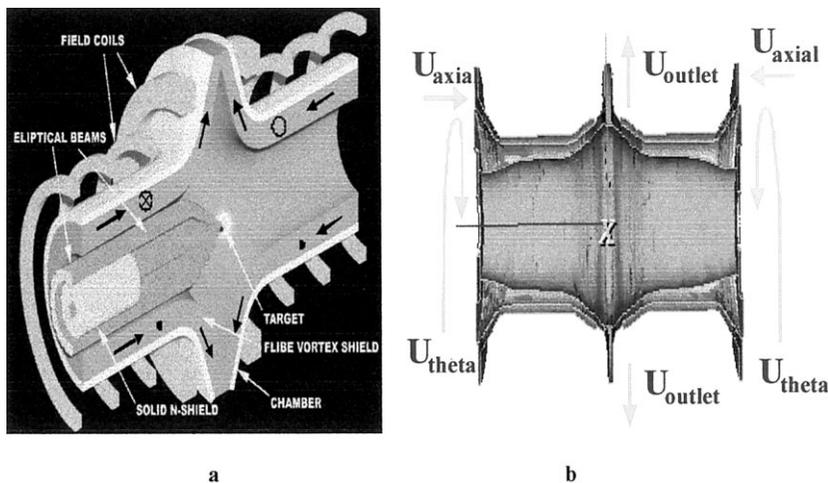


Fig. 6. (a) Preliminary layout of HIF concept with utilization of swirl idea. (b) A cross-section of 3-D fluid distribution in HIF chamber.

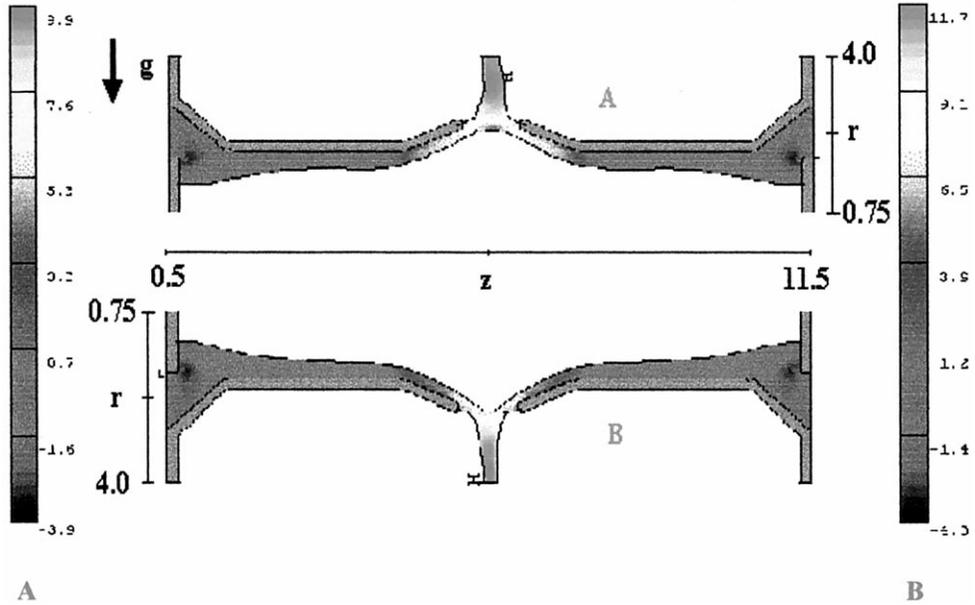


Fig. 7. Radial velocity distribution in r - z plane and liquid layer height distribution in the axial direction at an azimuthal angle where (A) the gravitational acceleration is in the opposite direction with the centrifugal acceleration; (B) the gravitational acceleration is in the same direction with the centrifugal acceleration.

liquid layer thickness along the chamber. As an example, the ratio of turbulent boundary layer thickness ($\delta = 0.16x^{6/7} (\rho U/\mu)^{-1/7}$) to 0.9-m liquid layer thickness for FLiBe at 550°C with 12 m/s velocity at a location of 10 m away from the inlet is 0.17. Therefore, potential flow theory can be used for the characterization of azimuthal flow for linear stability analysis. A 2-D (r - θ) linear stability analysis is performed using an irrotational velocity profile with surface tension, gravitational acceleration and the centrifugal acceleration models. Assumptions are: (1) boundary layer thickness is small compared to the free stream thickness; (2) only azimuthal velocity components are taken into account; (3) initial infinitely small perturbations are introduced for surface displacement and velocity potential. The system equations are: Laplacian of velocity potential and Bernoulli Equation with surface tension, centrifugal acceleration and gravitational acceleration models for a finitely small disturbance in the surface perturbation and velocity potential are expressed respectively as,

$$\nabla^2 \phi = 0 \quad (3)$$

$$\begin{aligned} \frac{\partial \phi_0}{\partial t} + \frac{\partial \phi'}{\partial t} + \frac{1}{2} (\nabla \phi_0)_\theta^2 + (\nabla \phi_0 \nabla \phi')_\theta \\ = \frac{1}{\rho} \left(\rho g \sin \theta \zeta - \frac{\sigma}{R_2} + \frac{\sigma}{R_2^2} \frac{\partial^2 \zeta}{\partial \theta^2} - \rho \frac{U_s^2 \zeta}{R_2} \right) \end{aligned} \quad (4)$$

The boundary condition at the boundary layer surface close to the back wall and free surface [12] are respectively,

$$V = \frac{\partial \phi}{\partial r} \hat{r}_0 = 0, \quad r = R_1 \quad (5)$$

$$\frac{\partial \phi}{\partial r} = \frac{D\zeta}{Dt} = \frac{\partial \zeta}{\partial t} + (\nabla \phi)_r \frac{\partial \zeta}{\partial r} + (\nabla \phi)_\theta \frac{1}{r} \frac{\partial \phi}{\partial \theta} \quad (6)$$

Since the set of partial differential equations has constant coefficients independent of time and θ , the method of normal modes is used in the linearization where there are small arbitrary perturbations of the form $\zeta, \phi' = (\hat{\zeta}, \hat{\phi}) e^{ik\theta + st}$. The coefficient of small disturbances in the exponent form is derived as,

$$S_{1,2} \cong \frac{U_s}{R_2} ik \pm i \sqrt{\frac{k}{R_2} \frac{1 - \left(\frac{R_1}{R_2}\right)^{2k}}{1 + \left(\frac{R_1}{R_2}\right)^k}} \sqrt{g \sin \theta - \frac{U_s^2}{R_2} - \frac{\sigma k^2}{R_2}} \quad (7)$$

Using the above equation, the liquid layer may be stable when gravitational acceleration, centrifugal acceleration and surface tension are taken into account for the condition of

$$g \sin \theta < \frac{U_s^2}{R_2} + \frac{\sigma k^2}{R_2} \quad (8)$$

The above result suggests that the free-vortex swirl flow is always stable when the azimuthal velocity component is taken into account since centrifugal acceleration should be more than gravitational acceleration in order for the liquid layer to adhere to the wall. Gravitational acceleration can have both stabilizing and destabilizing effect on the flow depending on the direction of the flow with respect to direction of the gravitational acceleration. Surface tension has a stabilizing effect for high wave numbers (short wave wavelength) as expected.

Hydrodynamic stability of the boundary layer on the concave back wall surface may not be an important issue since the boundary layer thickness is expected to be less than $\sim 17\%$ of the total thickness at its maximum point for the FRC chamber. However, there may be randomly existing Gortler vortices in the turbulent boundary layer [13–15]. Although large-scale inflows and outflows have a strong influence on the flow structure in the near wall region, near wall profiles of Reynolds-averaged quantities may show relatively minor differences between the flat and concave back wall cases.

There are also additional physical mechanisms that may effect the performance of the liquid layer by means of free surface heat transfer and swirl decay. These mechanisms are out of the scope of the present study and may be subject to future experimental work. As an example, evaluation of radial distribution of turbulence intensity in free-vortex along the axial direction and its effect on

swirl decay. In addition, evaluation of wave formations due to the relaxation of the boundary layer on the liquid layer surface leaving the inlet section is related to upstream effects such as relaminarization of the flow in the nozzle or the swirl generating inlet design (nozzle contraction ratio, upper wall surface curvature, etc.) [16,17] which are subjects of future work.

5. Conclusion and future work

Preliminary computational hydraulic analysis using a 3-D RANS solver with a free surface tracking algorithm predicts that a swirling liquid flow inside a cylindrical chamber for FRC/HIF configurations and inside a quasi-spherical chamber for the ST configuration may potentially form thick liquid walls to replace the customary first wall/blanket system. Parametric computational study indicated that the liquid layer thickness in the axial and azimuthal directions is strongly dependent on inlet axial and azimuthal velocity values and gravitational acceleration. A uniform liquid layer thickness in both the azimuthal and axial directions can be maintained for axial and azimuthal inlet velocities of 11 m/s and 13 m/s in a cylindrical chamber with 2 m radius and 12 m length. The swirling liquid wall idea is applied successfully to the ST (by modifying outboard back-wall topology) and HIF (by optimizing the distance between the two cylindrical chambers) configurations. A 2-D linear stability analysis using potential flow theory suggests that the mainstream flow is stable when surface tension, gravitational acceleration and centrifugal acceleration are considered.

For future work, the design and key issues in obtaining an optimum swirl generating inlet and kinematic energy recovering outlet will be addressed for the general swirl concept. Hydrodynamic stability analysis will be extended to take into account the Coriolis force and the axial velocity component. Turbulence generation in the swirl and at the boundary, due to the swirling flow itself will be evaluated.

For the application of the swirl idea to the HIF configuration, the diverging cusp section (angle,

length), liquid wall operating conditions and the distance between two cylindrical subsections will be optimized to eliminate possible liquid droplet formations when two liquid walls approach each other from opposite sides. For the application of the swirl idea to the ST configuration, the topology of the outboard will be modified to minimize the inlet velocity requirement in order to maintain the fluid adherence to the wall.

References

- [1] R.W. Moir, Rotating liquid blanket for toroidal fusion reactor, *Fus. Eng. Des.* 5 (1987) 269–272.
- [2] R.W. Moir, Liquid first walls for magnetic fusion energy configurations, *Nucl. Fus.* 37 (4) (1997) 557–566.
- [3] M.A. Abdou, and the APEX Team, 1998. Exploring Novel High Power Density Concepts for Attractive Fusion Systems, *Fus. Technol.*
- [4] J.A. Hassberger, Stability of the FMIT High Speed Free Surface Liquid Jet Flowing Along a Curved Back Wall, *Proceedings of Tenth Symposium on Fusion Engineering*, vol. 2, 1983, pp. 1849–1853.
- [5] K. Gulec, Stability of High Velocity Liquid Lithium Jet Over a Concave Surface, *Proceedings of 4th ASME International Heat and Mass Transport Congress*, May, 1997, Brussels.
- [6] Flow-3D®, 1998 User's manual, Flow Science, Report FSI-98-00-1.
- [7] Logan, 1999, Private Communication.
- [8] D.F. Kelsall, A study of the motion of solid particles in a hydraulic cyclone, *Trans. Inst. Chem. Eng.* 30 (1952).
- [9] D. Bradley, *The Hydrocyclone*, first ed., Pergamon Press, London, 1965.
- [10] S. Wang, Z. Rusak, On the stability of an axisymmetric rotating flow in a pipe, *Phys. Fluids* 8 (4) (1996) 1007–1016.
- [11] P.M. Ligrani, et al., Flow phenomena in swirl chambers, *Exp. Fluids* 24 (1998) 254–264.
- [12] H. Lamb, *Hydrodynamic Stability*, Cambridge University Press, Cambridge, 1960, pp 8–9.
- [13] I Tani, Production of longitudinal vortices in the boundary layer along a concave wall, *J. Geophys. Res.* 67 (8) (1962) 3075–3080.
- [14] B.R. Ramaprian, B.G. Shivaprasad, The structure of turbulent boundary layers along mildly curved surfaces, *J. Fluid Mech.* 85 (2) (1978) 273–303.
- [15] P. Bradshaw, K.C. Muck, P.H. Hoffman, The effect of concave surface curvature on turbulent boundary layers, *J. Fluid Mech.* 161 (1985) 371–403.
- [16] C Brennen, Cavity surface wave patterns and general appearance, *J. Fluid Mech.* 44 (1) (1970) 33–49.
- [17] J.A Hassberger, Comparison between measured and predicted performance of a high speed free surface liquid jet flowing along a curved wall, *Nucl. Technol. Fus.* 4 (1983) 433–438.