EFFECTS OF IMPERFECT INSULATING COATINGS ON THE FLOW PARTITIONING BETWEEN PARALLEL CHANNELS IN SELF-COOLED LIQUID METAL BLANKETS

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ABSTRACT

Fully developed liquid-metal flow in a system of three straight rectangular ducts is investigated. The ducts are electrically coupled by common conducting walls covered with an imperfect insulating layer. A numerical model of magnetohydrodynamic (MHD) flow in the system is described. Since no additional assumptions, such as in the core-flow solution, have been made, this model can be used for the analysis of MHD flow in parallel ducts with nearly perfect insulating coating. Any orientation of the applied uniform magnetic field is possible. Electrical conductivities of the dividing and exterior walls, and of the insulating layers in individual channels can be varied independently, as well as characteristics of insulation imperfections in each channel. A restriction of equal pressure gradients in all ducts is imposed, and the flow partitioning between parallel channels is examined. Results of the numerical simulation of the influence of insulation imperfections on flow distribution and velocity profiles are presented.

I. INTRODUCTION

An effective insulation of the electrically conducting coolant channel walls leads to a drastic reduction of the magnetohydrodynamic pressure drop in self-cooled liquid metal blankets. To electrically decouple the liquid metal from the load-carrying walls, and, consequently, to reduce MHD pressure drop, it has been previously proposed to use a self-healing insulating coating on the duct walls. However, insulation properties of the coatings may be reduced during a fusion reactor operation due to corrosion processes, irradiation damage and thermomechanical stresses. The reliability of such coatings remains uncertain because of the lack of information on the kinetics of the in-situ healing of insulation imperfections. When the insulation properties are uniformly reduced on all or certain duct walls or when local cracks in the coating occur at, at least, two different duct walls. MHD pressure drop may significantly increase and velocity profiles may be altered compared with flows in perfectly insulated ducts.

Typically, there is a number of parallel channels arranged between the inlet and outlet manifolds of a blanket segment. Therefore, the pressure drop is identical for all these channels, while the flow rate in each channel depends on its individual flow resistance. In an array of parallel coolant ducts which are coupled by common electrically conducting walls, electric currents induced in one channel can enter the other channels and affect the flow structure in the whole system. If all of the ducts' walls were covered with perfect insulating layer, such electrical coupling would no longer exist. But if the layer resistivity is reduced, the leakage currents create an additional pressure drop with respect to the flow in electrically separated channels and result in unequal flow distribution among the coolant ducts. This may lead to local overheating in the channels with reduced flow rate. Even when the overall increase in the pressure drop is within the tolerable design limits, insulation imperfections in one of the coolant channels may have a serious negative effect on heat transfer because of the flow redistribution.

This paper treats fully developed flow in a system of three rectangular ducts coupled by common electrically conducting walls. Several researches have investigated MHD flows in arrays of parallel ducts with a strong uniform magnetic field applied parallel to one set of walls. All but one of these studies make the assumption that the wall conductance ratio c and the Hartmann number M satisfy the relation c >> M^{-2}. In this case, electric currents carried by the side layers are neglected. The asymptotic solution is valid for arbitrary wall conductance ratio of the side and dividing walls, but the top and bottom walls (which are perpendicular to the applied magnetic field) are assumed to be much better conductors than the Hartmann layers: c_{top}, c_{bottom}>>M^{-1}. Present analyses do not require any of the above
assumptions. Unlike in the previous investigations, the walls are covered with nearly perfect insulating layer. All boundary layers are included and directly resolved. It allows modeling of the fully developed flows in which side-layer phenomena dominate (due to reduced resistance of the insulating coating or an external magnetic field which is inclined with respect to the side walls). A 2-D MHD flow code, which has been previously utilized for modeling of the flow in a single rectangular duct, has been upgraded in order to provide capabilities for numerical simulation of multiple-duct flows.

II. PROBLEM FORMULATION

The dimensionless equations governing the steady flow of a viscous, incompressible, conducting fluid in a straight channel of infinite length are

\[
\frac{dp^*}{dz} = \frac{1}{M^2} \left( \frac{\partial \Phi^*}{\partial x} - \frac{1}{\sigma_f} \left( \frac{B_y}{\sigma} \frac{\partial v^*}{\partial x} + B_x \frac{\partial v^*}{\partial y} \right) \right) - v^* \left( \frac{B_x^2}{2} + B_y^2 \right)
\]

\[
\frac{\sigma}{\sigma_f} \left( \frac{\partial v^*}{\partial x} - \frac{1}{\sigma_f} \left( \frac{B_y}{\sigma} \frac{\partial v^*}{\partial x} + B_x \frac{\partial v^*}{\partial y} \right) \right) = 0.
\]

(1)

Here v is the fluid velocity, \( \Phi \) is the electric potential, \( B = B_y(\cos \alpha) \hat{e}_x + B_x(\sin \alpha) \hat{e}_y \) is a uniform external magnetic field of arbitrary orientation, \( M = aB_0 \sqrt{\sigma_f/\mu} \) is the Hartmann number, \( \sigma \) denotes fluid, wall, insulator or crack conductivity, depending on the location of the point at which equations (1) are solved. Nondimensional parameters are defined using the characteristic scale length \( a \), fluid conductivity \( \sigma_f \), magnetic field \( B_0 \) and \( v_0 \) (characteristic fluid velocity which will be defined later):

\[
x^* = \frac{x}{a}, \quad B^* = \frac{B}{B_0}, \quad v^* = \frac{v}{v_0},
\]

\[
p^* = \frac{P}{\nu_0 B_0^2 \sigma_f a}, \quad \phi^* = \frac{\phi}{\nu_0 B_0 a}.
\]

(2)

Consider fully developed MHD flow in a system of three straight rectangular channels (see Figure 1). All channels have the same width 2a (in the x-direction) and the same height 2b (in the y-direction). The point \( x^*=0, \ y^*=0 \) is located at the center of channel 2. The problem is not symmetrical with respect to \( y^*=0 \) because the top and bottom walls may have different conductivities and/or thicknesses, and the flow might not be symmetrical, depending on the location of insulation imperfections. That is why a solution is sought in the whole duct. The boundary conditions include zero velocity at the walls, zero current out of the duct (\( \nabla \Phi \cdot n = 0 \) at the surface). Continuity of the potential is imposed at the interfaces between materials of different electrical conductivity, hence no thin conducting wall approximation is necessary. The equations (1) are solved numerically, using a finite element method, for the velocity \( v \) and electric potential \( \Phi \), with a given pressure gradient (in each channel), magnetic field strength and direction. This paper treats the case of equal pressure gradients in all channels. The characteristic fluid velocity \( v_0 \) is defined as the average velocity in the whole duct:

![Figure 1. Schematic diagram of the flow in three rectangular ducts.](image-url)
where \( Q^t = Q_1 + Q_2 + Q_3 \) is the total flow rate.

The code can be used to model MHD flows with very high Hartmann numbers (successfully tested for \( M = 10^3 \)), since non-uniform grids, which provide enough data points in the boundary layers and at the vicinity of the local insulation imperfections, are employed. Electrical conductivities of the walls \( \sigma_w^{i} \) (i=1,...,6), insulating layers \( \sigma_{ins}^{i} \) (i=1, 2, 3) and insulation imperfections \( \sigma_c \) can be varied independently, as well as the wall thicknesses \( t_w^{i} \) and the thickness \( t_e \) and width \( \delta_c \) of insulation defects.

For all cases discussed in this paper, we consider square channels \( a=b \), all six walls have the same thickness \( t_e/a=0.025 \) and the same electrical conductivity \( \sigma_w/\sigma_e=4 \), which yields the wall conductance ratio \( c=0.1 \). The insulating layer thickness \( t_{ins}/a = t_e/a = 10^{-4} \).

III. RESULTS AND DISCUSSION

Described below are numerical modeling results for various combinations of insulating layer resistances, insulation imperfection (crack) parameters and different orientation of the applied magnetic field. The insulating layer resistance will be characterized by the following parameter:

\[
\theta = \frac{\rho_{ins} a}{2bM/\sigma_e} \quad (4)
\]

which is the ratio of the insulating coating resistance to the Hartmann layer resistance. It will be used to describe the resistance of the entire coating in one (or all) of the channels or the resistance of the coating on certain walls of the channels. Insulation imperfections (cracks) in the coating are treated as parts of the coating with reduced electrical resistivity. They will be characterized by

\[
\kappa = \frac{\rho_c a}{\rho_e \delta_c 2bM} \quad (5)
\]

which is the ratio of the crack resistance to the Hartmann layer resistance. The term "perfectly insulated duct" will be used to describe the situation when the layer resistance is so high that the MHD pressure drop and velocity profile are the same as in the duct with nonconducting walls (in this case \( \theta \gg 1 \)), while "a highly conducting crack" corresponds to \( \kappa \ll 1 \).

This paper does not discuss any cases when the applied magnetic field is not parallel to one set of walls.

A. Insulating Layer Resistances Vary Simultaneously in All Three Ducts.

If the applied magnetic field is aligned with the top and bottom wall (see Figure 1), i.e. \( B_x=B_y=0 \), then the flow rates in all channels remain equal for any \( \theta \). But if it is parallel to the dividing walls, i.e. \( B_x=0 \), \( B_y=B_0 \), then the flow rate in the outer channels is somewhat higher than in the middle one, when the layer resistance is low. A maximum flow rate imbalance \( (Q_3-Q_2)/Q_2 \) is 10% \( (M=10^3) \) for bare conducting walls. In this case, the flow is of Hartmann type in the middle channel, while in the outer channels near-wall jets at the exterior side walls are present (see Figure 2). These results match the analytical solution for the case of equal pressure gradients in all three channels. Figure 3 shows velocity profiles in channels 2 and 3 for \( \theta=10^{-3} \). Velocity profile in channel 2 (Figure 3(a)) is similar to the one that arises in a single duct and is characterized by the presence of thick jets (compared to side-laver jets in a single duct with bare conducting walls) at the side walls. A side-layer jet at the right wall of channel 3 (Figure 3(b)) resembles the one in channel 2. But this jet is not flat in the core region: there it looks more like a side-layer jet in a single duct with bare conducting walls.

B. Insulating Layer Resistance Varies in the Middle Channel.

Consider the case of varying layer resistance in channel 2, while channels 1 and 3 remain perfectly insulated. Variation of the flow rate with \( \theta \) for various Hartmann numbers is shown in Figure 4. Once the insulating layer resistance drops below the Hartmann layer resistance \( \theta < 1 \), the flow rate starts decreasing quickly, and it is less than 5% of the total \( (\theta = 10^3 - 10^2) \) when \( \theta < 0.05 \). At the same time, the average velocities in the outer channels increase. Then, if the rate of coating self-healing in channel 2 is not sufficient to maintain \( \theta = 1 \), this may promote further degradation of the coating, due to thermal stress.

C. Local Insulation Imperfection and Reduced Insulating Layer Resistance at Certain Walls in Channel 2.

Consider the case of the applied magnetic field aligned with the top and bottom walls \( (B_x=B_y=0) \). If we assume that \( B_y \) corresponds to the toroidal magnetic field of a fusion reactor, then the insulating layer on either top or bottom wall is more likely to be damaged, since it is located closer to the plasma. If one of the side walls in any (or all) of the channels looses its insulating coating, the MHD pressure drop does not increase because there is no closed path for the electric current through the conducting walls. If electrical conductivity of the insulating layer on the top wall of channel 2 is equal to \( \sigma_e \),
(this corresponds to $\theta = 5 \times 10^{-9}$ for $M = 10^4$) and there is an insulating imperfection with $\sigma = \sigma_f$ at the bottom wall, in the right corner (from $x^* = 0.9998$ to $x^* = 1$), a large negative-velocity jet forms in the vicinity of the crack, while the average velocity in the duct is very small (see Figure 5(a)). Now if the insulating layer on the left wall of channel 2 also decreases to $\theta = 5 \times 10^{-9}$, it does not cause any further significant flow redistribution, but the velocity profile changes (as shown in Figure 5(b)). The amount of flow in the negative-velocity jet decreases by an order of magnitude. Electric currents paths at the bottom wall (except in the vicinity of the crack) then look similar to the case of bare conducting wall duct.

D. Varying Insulating Layer Resistance at the Dividing Wall Between Channels 1 and 2.

The applied magnetic field is again aligned with the top and bottom walls. Electrical conductivity of the insulating layer on the left wall of channel 2 is constant and equal to $\sigma_f$ (then $\theta = 5 \times 10^{-9}$ for $M = 10^4$). Figure 6 shows how the flow distribution changes if the insulating layer resistance on the other side of this dividing wall (which is the right wall of channel 1) decreases. At first, the flow rate in channel 2 is only 0.4% of the total, while channels 1 and 3 carry equal amounts of flow. But as $\theta$ decreases, the flow rate in channel 1 is reduced to that in

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**Figure 2.** Fluid velocity at $y^* = 0$ in the right half of channel 2 and in channel 3 (the profile is symmetrical with respect to $x^* = 0$) for $M = 10^3$ and $B_x = 0$, $B_y = B_0$. The insulating layer resistance is the same in all three channels.

**Figure 3 (a, b).** Velocity profiles in channel 2 (a) and channel 3 (b) for $M = 10^3$ and $B_x = 0$, $B_y = B_0$. The insulating layer resistance is the same in all three channels: $\theta = 10^{-3}$. 

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channel 2. When $\theta=5 \times 10^{-9}$ on both sides of wall 4, 98% of the flow is carried by channel 3.

![Graph](image)

Figure 4. Variation of the flow rate in channel 2 with the insulating layer resistance in that channel. Perfect insulating layers in channels 1 and 3.

IV. CONCLUSIONS

A numerical model of fully developed MHD flow in the system of three straight rectangular ducts is described. The ducts are electrically coupled by common conducting walls covered with imperfect insulating layers. Characteristics of the insulating layer and insulation imperfections can be varied independently in each duct. The restriction of equal pressure gradients in all channels is imposed, and the flow partitioning between the parallel channels is examined. Four different cases are treated, to show the capabilities of the code. The influence of the Hartmann number, insulating layer resistance and insulation imperfection resistance on flow distribution and velocity profiles is investigated.

If insulating layer resistances vary simultaneously in all three ducts, the maximum flow rate imbalance is 10% ($M=10^3$) for bare conducting walls. The flow rate in the outer channels is higher than in the middle one. It occurs only when the applied magnetic field is parallel to the dividing walls. Once the insulating layer resistance in one of the channels drops below the Hartmann layer resistance, the flow rate in that channel starts decreasing quickly, and it is less than 5% of the total when $\theta$ is less than 0.05. Even if the insulating coating is seriously damaged on both sides of just one of the dividing walls, the flow rates in the adjacent channels may be reduced to less than 1% of the total (for $M=10^4$) and 98% of the flow is then carried by the third channel.

If insulation imperfections do not heal fairly quickly, they may induce further degradation of the insulating layer in the same channel and in the adjacent channels.

![Graph](image)

Figure 5 (a, b). Velocity profile in channel 2 for $M=10^4$ and $B_x = B_0$, $B_y = 0$. Perfect insulating layers in channels 1 and 3. $\theta=5 \times 10^{-9}$ for the insulating layer on the top wall of channel 2. Highly conducting crack ($\kappa=5 \times 10^5$) at the bottom right corner of channel 2 (width $5c/a=2 \times 10^{-4}$). The insulating layer resistance on the left wall of channel 2: (a) $\theta=5 \times 10^2$; (b) $\theta=5 \times 10^{-9}$. 

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Figure 6. Variation of the flow rates in different channels with the layer resistance on the right wall of channel 1 for M=10^4, B_x=B_y=B_z=0 and constant θ = 5 × 10^{-9} for the layer on the left wall of channel 2. Perfect insulating layer in channel 3.

due to flow redistribution and local overheating in channels with reduced flow rates. Thus, when determining the rate of self-healing which is required in order to maintain an equilibrium between generation and healing of the imperfections, one should take this factor into consideration.

REFERENCES


