Introduction to MHD and Applications to Thermofluids of Fusion Blankets

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Introduction to MHD and Applications to Thermofluids of Fusion Blankets

OUTLINE

• MHD* basics
• MHD and liquid blankets
• UCLA activities in thermofluid MHD

* Our focus is incompressible fluid MHD. Don’t mix with Plasma Physics.
MHD basics

• What is MHD?
• MHD applications
• Magnetic fields
• Electrically conducting fluids
• MHD equations
• Scaling parameters
• Hartmann problem
• MHD flow in a rectangular duct
• MHD pressure drop
• Electric insulation
• Complex geometry / non-uniform B-field
• Numerical simulation of MHD flows
What is MHD?

MHD covers phenomena in electrically conducting fluids, where the velocity field $\mathbf{V}$, and the magnetic field $\mathbf{B}$ are coupled.

- Any movement of a conducting material in a magnetic field generates electric currents $\mathbf{j}$, which in turn induces a magnetic field.

- Each unit volume of liquid having $\mathbf{j}$ and $\mathbf{B}$ experiences MHD force $\sim \mathbf{j} \times \mathbf{B}$, known as the “Lorentz force”.

In MHD flows in blanket channels, interaction of the induced electric currents with the applied plasma-confinement magnetic field results in the flow opposing Lorentz force that may lead to high MHD pressure drop, turbulence modifications, changes in heat and mass transfer and other important MHD phenomena.
A few facts about MHD

- **Alfvén** was the first to introduce the term “MAGNETOHYDRODYNAMICS”. He described astrophysical phenomena as an independent scientific discipline.

- The official birth of incompressible fluid Magnetohydrodynamics is 1936-1937. **Hartmann** and **Lazarus** performed theoretical and experimental studies of MHD flows in ducts.

- The most appropriate name for the phenomena would be “MagnetoFluidMechanics,” but the original name “Magnetohydrodynamics” is still generally used.

**Hannes Alfvén** (1908-1995), winning the Nobel Prize for his work on Magnetohydrodynamics.
MHD applications, 1

- **Astrophysics** (planetary magnetic field)
- **MHD pumps** (1907)
- **MHD generators** (1923)
- **MHD flow meters** (1935)
- **Metallurgy** (induction furnace and casting of Al and Fe)
- **Dispersion (granulation) of metals**
- **Ship propulsion**
- **Crystal growth**
- **MHD flow control** (reduction of turbulent drag)
- **Magnetic filtration and separation**
- **Jet printers**
- **Fusion reactors** (blanket, divertor, limiter, FW)

A snapshot of the 3-D magnetic field structure simulated with the Glatzmaier-Roberts geodynamo model. Magnetic field lines are blue where the field is directed inward and yellow where directed outward. One year of computations using a supercomputer! *Nature*, 1999.
In some MHD applications, the electric current is applied to create MHD propulsion force.

An electric current is passed through seawater in the presence of an intense magnetic field. Functionally, the seawater is then the moving, conductive part of an electric motor, pushing the water out the back accelerates the vehicle.

The first working prototype, the *Yamato 1*, was completed in Japan in 1991. The ship was first successfully propelled 1992. Yamato 1 is propelled by two MHD thrusters that run without any moving parts.

In the 1990s, Mitsubishi built several prototypes of ships propelled by an MHD system. These ships were only able to reach speeds of 15km/h, despite higher projections.
Magnetic fields

- **Earth** – $0.5 \times 10^{-4}$ T
- **Sun** – $10^{-4}$ T, up to $0.4$ T at sunspots
- **Jupiter** – $10^{-2}$ T (strongest planetary magnetic field in the solar system)
- **Permanent laboratory magnets** with $\sim 0.1$ m gap – about 1-2 T
- **Electromagnets** – 25-50 T
- **Fusion Reactor (ARIES RS)** – 12 T
- **Experimental Fusion Reactor (NSTX)** – 1.5 T

Supplying power to the world's strongest long-pulse magnet at Los Alamos' National High Magnetic Field Laboratory is a 1.4 billion-watt generator, itself the largest among magnetic power sources. It can produce enough energy to power the entire state of New Mexico.
**Electrically conducting fluids**

<table>
<thead>
<tr>
<th>Liquid</th>
<th>$\sigma^*$, 1/Ohm×m</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weak electrolytes</strong></td>
<td>10^{-4} to 10^{-2}</td>
</tr>
<tr>
<td>Water+25% NaCl (20°C)</td>
<td>21.6</td>
</tr>
<tr>
<td>Pure H$_2$SO$_4$ (20°C)</td>
<td>73.6</td>
</tr>
<tr>
<td><strong>Strong electrolytes</strong></td>
<td>10^1 to 10^2</td>
</tr>
<tr>
<td>Molten salts (FLiNaBe, FLiBe at 500°C)</td>
<td>~ 150</td>
</tr>
<tr>
<td><strong>Liquid metals</strong></td>
<td>10^6 to 10^7</td>
</tr>
<tr>
<td>Mercury (20°C)</td>
<td>1.0×10^6</td>
</tr>
</tbody>
</table>

*$\sigma$, electrical conductivity (1/Ohm×m), shows ability of liquid to interact with a magnetic field
MHD equations

- Navier-Stokes equations with the Lorentz force
  \[
  \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V} + \mathbf{g} + \frac{1}{\rho} \mathbf{j} \times \mathbf{B}
  \]  
  (1)

- Continuity
  \[
  \nabla \cdot \mathbf{V} = 0
  \]  
  (2)

- Energy equation with the Joule heating
  \[
  \rho C_p \left( \frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T \right) = k \nabla^2 T + \frac{j^2}{\sigma} + q''''
  \]  
  (3)

- Ampere’s law
  \[
  \mathbf{j} = \mu^{-1} \nabla \times \mathbf{B} \quad \text{(vacuum: } \mu_0 = 4\pi \times 10^{-7} = 1.257 \times 10^{-6} \text{ } \text{H/m)}
  \]  
  (4)

- Faraday’s law
  \[
  \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
  \]  
  (5)

- Ohm’s law*
  \[
  \mathbf{j} = \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B})
  \]  
  (6)

*Eqs.(4-6) are usually grouped together to give either a vector induction equation or a scalar equation for electric potential
Basic scaling parameters

**Reynolds number**

\[ \text{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{U_0 L}{v} \]

**Magnetic Reynolds number**

\[ \text{Re}_m = \frac{\text{Convection of } B}{\text{Diffusion of } B} = \frac{\text{Induced field}}{\text{Applied field}} = \frac{U_0 L}{\nu_m} = \mu_0 \sigma U_0 L \]

**Hartmann number**

\[ Ha = \left( \frac{\text{Electromagnetic forces}}{\text{Viscous forces}} \right)^{1/2} = B_0 L \sqrt{\frac{\sigma}{\nu \rho}} \]

**Alfven number**

\[ Al = \frac{N}{\text{Re}_m} = \frac{B_0^2}{\mu_0 \rho U_0^2} = \frac{\text{Magnetic field energy}}{\text{Kinetic energy}} \]

**Stuart number (interaction parameter)**

\[ N \equiv St = \frac{Ha^2}{\text{Re}} = \frac{\sigma B_0^2 L}{\rho U_0} = \frac{\text{Electromagnetic forces}}{\text{Inertia forces}} \]

**Batchelor number (magnetic Prandtl number)**

\[ Bt \equiv \text{Pr}_m = \frac{\text{Re}_m}{\text{Re}} = \mu_0 \sigma v = \frac{v}{\nu_m} \]
Hartmann problem, 1


- Fundamental MHD problem. MHD analog of plane Poiseuille flow.
- Classic formulation (J. Hartmann, 1937) addressed fully developed flow in a rectangular duct with a large aspect ratio, $a/b >> 1$.
- Mathematically, the problem reduces to two coupled 2-d order ODEs, solved analytically.
If $Ha$ grows, the velocity profile becomes more and more flattened. This effect is known as the "Hartmann effect".

The thin layer near the wall where the flow velocity changes from zero to $U_m$ is called the "Hartmann layer".

The Hartmann effect is caused by the Lorentz force, which accelerates the fluid in the Hartmann layers and slows it down in the bulk.
MHD flow in a rectangular duct, 1


Fully developed flow equations (dimensionless):

\[
\begin{align*}
\frac{\partial^2 U^*_y}{\partial z^*_y} + \chi^2 \frac{\partial^2 U^*_y}{\partial y^*_y} & + \text{Ha} \left( \frac{\partial B^*_z}{\partial z^*_y} \cos \alpha + \chi \frac{\partial B^*_z}{\partial y^*_y} \sin \alpha \right) + 1 = 0 \\
\frac{\partial^2 B^*_y}{\partial z^*_y} + \chi^2 \frac{\partial^2 B^*_y}{\partial y^*_y} & + \text{Ha} \left( \frac{\partial U^*_z}{\partial z^*_y} \cos \alpha + \chi \frac{\partial U^*_z}{\partial y^*_y} \sin \alpha \right) = 0
\end{align*}
\]

Boundary conditions:

\[
\begin{align*}
z^*_z = \pm 1: & \quad U^*_z = 0, \quad c_w \frac{\partial B^*_z}{\partial z^*_y} \pm B^*_z = 0 \\
y^*_y = \pm 1: & \quad U^*_y = 0, \quad c_w \chi \frac{\partial B^*_y}{\partial y^*_y} \pm B^*_y = 0
\end{align*}
\]

Dimensionless parameters:

\[
\text{Ha} = B_0 b \sqrt{\frac{\sigma}{\nu \rho}} \quad \text{(Hartmann number)}
\]

\[
c_w = \frac{t_w \sigma_w}{b \sigma} \quad \text{(wall conductance ratio)}
\]

\[
\chi = b/a \quad \text{(aspect ratio)}
\]
MHD flow in a rectangular duct, 2

Duct with insulating walls ($c_w=0$). Induced magnetic field

$Ha=600$, $c_w=0$, $\chi=2$, $\alpha=0$

Electric currents induced in the flow bulk close their circuit in the thin Hartmann layers at the duct walls perpendicular to the applied magnetic field.
MHD flow in a rectangular duct, 3

Duct with insulating walls ($c_w=0$). Velocity

- The velocity profile is flattened in the bulk. High velocity gradients appear near the walls.

- At the walls perpendicular to the B-field, two MHD boundary layers with the thickness $\sim 1/H_a$ are formed, called “Hartmann layers”.

- At the walls parallel to the magnetic field, there are two secondary MHD boundary layers with the thickness $\sim 1/H_a^{0.5}$, called “side layers”.

![Velocity profile diagram](image-url)
MHD flow in a rectangular duct, 4

Duct with conducting walls ($c_w > 0$). Induced magnetic field

Much stronger electric currents are induced compared to the non-conducting duct. The currents close their circuit through the walls.
High-velocity jets appear near the walls parallel to the B-field. The velocity profile is called “M-shaped”.

The jet formation occurs due to high flow-opposing vortical Lorentz force in the bulk, while no force appears near the parallel walls.

The M-shaped profile has inflection points. Under certain conditions, the flow becomes unstable.
MHD pressure drop

Hartmann flow

Hartmann layer $\delta_{Ha} \approx b/Ha$

$\Delta p = \frac{\lambda}{2b} \frac{l}{2} \frac{\rho U_m^2}{2}$.

$\lambda$ is the pressure drop coefficient

If $Ha \rightarrow 0$, $\lambda \rightarrow \lambda_0 = 24/Re$.

$\lambda = \frac{8}{\text{Re}} \frac{Ha^2}{c_w + 1} \frac{c_w Ha + \tanh Ha}{Ha - \tanh Ha}$, $\text{Re} = \frac{U_m 2b}{\nu}$

$\frac{\lambda}{\lambda_0} = \frac{1}{3} \frac{Ha^2}{c_w + 1} \frac{c_w Ha + \tanh Ha}{Ha - \tanh Ha}$

I. Non-conducting walls ($c_w = 0$), $Ha >> 1$:

$\frac{\lambda}{\lambda_0} = \frac{1}{3} Ha$

II. Conducting walls ($c_w > 0$), $Ha >> 1$:

$\frac{\lambda}{\lambda_0} = \frac{1}{3} \frac{c_w}{c_w + 1} Ha^2$

Conclusion: In electrically conducting ducts in a strong magnetic field, the MHD pressure drop is $\sim Ha^2$, while it is $\sim Ha$ in non-conducting ducts. LM blanket: $Ha \sim 10^4$!
Electrical insulation

• Either insulating coatings or flow inserts can be used for decoupling the liquid metal from the electrically conducting walls to reduce the MHD pressure drop.

• Even microscopic defects in the insulation will result in electrical currents closing through the walls.

• Challenge: Development of stable coatings with good insulation characteristics.

Current leakage through a microscopic crack in a 50 μm insulating coating

Electro-circuit analogy
Complex geometry / non-uniform magnetic field

• Complex geometry and non-uniform magnetic field MHD flows are similar in nature.

• The distinctive feature is 3-D (axial) currents, which are responsible for extra MHD pressure drop and M-shaped velocity profiles.

• Such problems are very difficult for analytical studies. Experimental and numerical data are available, showing

\[ \lambda_{3D} \sim N = Ha^2 / Re \]
Numerical simulation of MHD flows

Status of MHD code development

- A number of 2-D/3-D MHD computations were performed in 90’s based on the full set of the Navier-Stokes-Maxwell equations.
- These computations showed a limit on the Hartmann number caused by lack of charge conservation: $Ha=300-500$ ($10^4$ in the LM blanket applications!).
- Inertialess approaches (e.g. “core flow approximation”) were developed, capable of doing high $Ha$ computations. However, these approaches neglect many important phenomena related to convective terms.
- Challenge: Development of special numerical techniques particularly suited for high $Ha$ MHD.

MHD codes

- Numerous 2-D/3-D research codes
- FLUENT. MHD module based on implementation of $B$- or $\phi$-formulation. Tests at UCLA (2003) showed a limit on $Ha$ ($\sim10^1-10^2$). Moreover, results obtained with the $\phi$-formulation seem to be wrong.
- CFX. User-developed MHD module based on $\phi$-formulation: Coventry University (Molokov et. al, 2002) and FZK (Buhler et. al, 2004).
- FLOW 3D. User-developed MHD module based on $B$-formulation: UCLA (Huang et. al, 2002).
- Special MHD software is being developed by METAHEURISTICS (USA) and HyPerComp (USA) in collaboration with UCLA.
MHD and liquid Breeder blankets

- MHD issues of liquid Breeder blankets
- Examples of MHD calculations in liquid Breeder blankets
MHD issues of liquid blankets, 1

Liquid blanket designs have the best potential for high power density, but MHD interactions of the flowing liquid with the confinement B-field may lead to:
- extreme MHD drag resulting in high blanket pressure and stresses, and flow balance disruption
- velocity profile and turbulence distortion resulting in severe changes in heat transfer, corrosion and tritium transport

MHD effects are specific to the blanket design. In self-cooled liquid metal blankets, the MHD pressure drop is considered as the main issue, while in self-cooled molten salt blankets, the blanket performance depends on the degree of turbulence suppression by a magnetic field.
MHD issues of liquid blankets, 2

<table>
<thead>
<tr>
<th>MHD issue</th>
<th>Self-cooled LM blanket</th>
<th>Dual-coolant LM blanket</th>
<th>He-cooled LM blanket</th>
<th>MS blanket</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MHD pressure drop and flow distribution</strong></td>
<td>Very important</td>
<td>Important</td>
<td>Inlet manifold ?</td>
<td>Not important</td>
</tr>
<tr>
<td><strong>Electrical insulation</strong></td>
<td>Critical issue</td>
<td>Required for IB blanket</td>
<td>Not needed ?</td>
<td>Not needed</td>
</tr>
<tr>
<td><strong>MHD turbulence</strong></td>
<td>Cannot be ignored</td>
<td>Important. 2-D turbulence</td>
<td>Not applicable</td>
<td>Critical issue (self-cooled)</td>
</tr>
<tr>
<td><strong>Buoyancy effects</strong></td>
<td>Cannot be ignored</td>
<td>Important</td>
<td>May effect tritium transport</td>
<td>Have not been addressed</td>
</tr>
<tr>
<td><strong>MHD effects on heat transfer</strong></td>
<td>Cannot be ignored</td>
<td>Important</td>
<td>Not important</td>
<td>Very important (self-cooled)</td>
</tr>
</tbody>
</table>
### Key DCLL parameters in three blanket scenarios

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DEMO</th>
<th>ITER H-H</th>
<th>ITER D-T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface heat flux, (Mw/m^2)</td>
<td>0.55</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Neutron wall load, (Mw/m^2)</td>
<td>3.08</td>
<td>-</td>
<td>0.78</td>
</tr>
<tr>
<td>PbLi In/Out T, °C</td>
<td>500/700</td>
<td>470/~450</td>
<td>360/470</td>
</tr>
<tr>
<td>2a x 2b x L, (m)</td>
<td>0.22x0.22x2</td>
<td>0.066x0.12x1.6</td>
<td>0.066x0.12x1.6</td>
</tr>
<tr>
<td>PbLi velocity, (m/s)</td>
<td>0.06</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Magnetic field, (T)</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>Re</strong></td>
<td>61,000</td>
<td></td>
<td>30,500</td>
</tr>
<tr>
<td><strong>Ha</strong></td>
<td>11,640</td>
<td></td>
<td>6350</td>
</tr>
<tr>
<td><strong>Gr</strong></td>
<td>(3.52\times10^{12})</td>
<td></td>
<td>(7.22\times10^9)</td>
</tr>
</tbody>
</table>

*Velocity, dimensions, and dimensionless parameters are for the poloidal flow. DEMO parameters are for the outboard blanket.*
### Summary of MHD pressure drops for ITER TBM

<table>
<thead>
<tr>
<th>Flow</th>
<th>$\Delta P_i, \text{MPa}$</th>
<th>$\Delta P_i/\Delta P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Front channel</td>
<td>$0.384 \times 10^{-3}$</td>
<td>0.13</td>
</tr>
<tr>
<td>2. Return channel</td>
<td>$0.485 \times 10^{-3}$</td>
<td>0.16</td>
</tr>
<tr>
<td>3. Concentric pipe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(internal, uniform B-field)</td>
<td>$15.4 \times 10^{-3}$</td>
<td>5.1</td>
</tr>
<tr>
<td>4. Concentric pipe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(annulus, uniform B-field)</td>
<td>$0.0286$</td>
<td>9.5</td>
</tr>
<tr>
<td>5. Concentric pipe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(internal, fringing B-field)</td>
<td>$0.0585$</td>
<td>19.3</td>
</tr>
<tr>
<td>6. Concentric pipe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(annulus, fringing B-field)</td>
<td>$0.0585$</td>
<td>19.3</td>
</tr>
<tr>
<td>7. Inlet manifold</td>
<td>$0.070-0.140$</td>
<td>23.2</td>
</tr>
<tr>
<td>8. Outlet manifold</td>
<td>$0.070-0.140$</td>
<td>23.3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$0.302-0.442$</td>
<td>100</td>
</tr>
</tbody>
</table>
Parametric analysis was performed for poloidal flows to access FCI effectiveness as electric/thermal insulator.

- $\sigma_{\text{SiC}} = 1\text{-}500 \text{ S/m}$, $k_{\text{SiC}} = 2\text{-}20 \text{ W/m-K}$
- Strong effect of $\sigma_{\text{SiC}}$ on the temperature field exists via changes in the velocity profile.
- FCI properties were preliminary identified: $\sigma_{\text{SiC}} \approx 100 \text{ S/m}$, $k_{\text{SiC}} \approx 2 \text{ W/m-K}$
UCLA activities in thermofluid MHD

- UCLA MHD group
- MHD Lab at UCLA
- Code development: HIMAG
- Examples of R&D
- Examples of recent publications
UCLA MHD group is one of the world’s key teams working in the area of fusion MHD

- Blanket performance is strongly affected by MHD phenomena
- UCLA group performs MHD studies for liquid breeder blankets (with recent emphasis on DCLL conditions) for both DEMO blanket and ITER TBM
- The research addresses fundamental issues of complex geometry flows of electrically conducting fluids in strong reactor-type magnetic fields via:
  - computer simulations
  - experiments
  - model development

Research topics

- Blanket thermal hydraulics
- MHD flows in manifolds (experiment and modeling)
- Low conductivity fluid turbulent MHD flows (experiment and DNS)
- DNS of low/high conductivity fluid turbulent flows
- Development of turbulent closures for MHD flows in a strong magnetic field
- Buoyancy-driven MHD flows in vertical ducts (modeling)
MHD Lab at UCLA

MTOR facilities

JUPITER 2 MHD Heat Transfer Exp. in UCLA FLIHY Electrolyte Loop

BOB magnet

QTOR magnet and LM flow loop
The HyPerComp Incompressible MHD Solver for Arbitrary Geometry (HIMAG) has been developed over the past several years by a US software company HyPerComp in collaboration with UCLA.

At the beginning of the code design, the emphasis was on the accurate capture of a free surface in low to moderate Hartmann number flows.

At present, efforts are directed to the code modification and benchmarking for higher Hartmann number flows in typical closed channel configurations relevant to the DCLL blanket.

High Hartmann number computations are now possible due to a novel numerical technique developed at UCLA.
Model development focuses on key MHD phenomena that affect thermal blanket performance via modification of the velocity field.

A. Formation of high-velocity near-wall jets

B. 2-D MHD turbulence in flows with M-type velocity profile

C. Reduction of turbulence via Joule dissipation

D. Natural/mixed convection

E. Strong effects of MHD flows and FCI properties on heat transfer
Experiments and numerical simulations are being conducted for prototypic blanket elements

Test section for studying flow distribution and MHD pressure drop in the inlet PbLi manifold

Modeling of flow development in the manifold experiment using HIMAG. The liquid metal enters the manifold through the feeding channel, passes the expansion section, and then further develops through three parallel channels.
Recent publications on MHD, 1

Recent publications on MHD, 2

- M.-J. Ni, et al., *Consistent and Conservative Scheme for MHD at Low Magnetic Reynolds Number based on a Strong Conservative Formula*, 7th World Congress on Computational Mechanics, Los Angeles, CA, July, 16-22, 2006
Recent publications on MHD, 3

Recent publications on MHD, 4