

**MODELLING AND ANALYSIS OF  
DT FUEL SELF-SUFFICIENCY  
IN FUSION REACTORS**

Mohamed A. Abdou  
UCLA

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## Purpose of Effort

- Understand and quantify conditions necessary to achieve DT self-sufficiency in fusion reactors
- Identify R&D areas in physics and engineering of critical importance to fuel self-sufficiency

## Approach

- Develop mathematical models to describe tritium behavior in all components in the fuel cycle
- Develop probability distributions for possible outcome of fusion physics and engineering R&D
- Perform sensitivity analysis to determine importance of parameters
- Estimate range of uncertainties
- Identify critical R&D needs for fuel self-sufficiency

## TRITIUM BREEDING PROBLEM

- A part of DT fuel self-sufficiency issue

- Self-sufficiency condition:

$\Lambda_r$  = Required tritium breeding ratio

$\Lambda_a$  = Achievable breeding ratio

$\Lambda_r > \Lambda_a$

- Key question:

Magnitude of uncertainties in  $\Lambda_r$ ,  $\Lambda_a$

- Conventional types of uncertainties
- Unconventional type

## REQUIRED TBR

$$\Lambda_r = 1 + G_o + \Delta_G$$

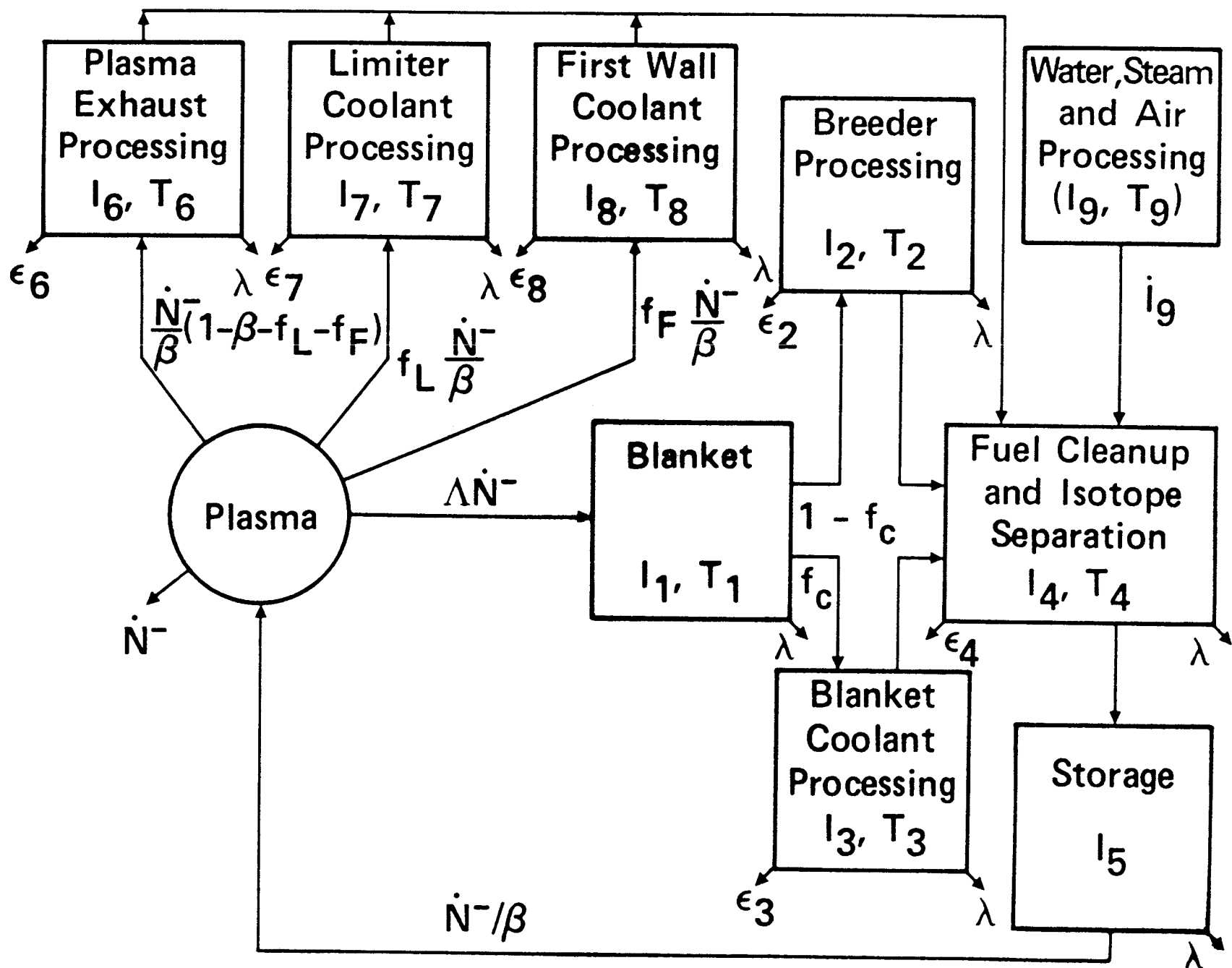
$G_o$  = doubling time margin for a reference conceptual design

$\Delta_G$  = uncertainty associated with G

### Model

- Model was formulated and used to evaluate dependence of  $\Lambda_r$  on reactor parameters.
- Methods for estimating  $\Delta_G$  are under development. Initial results are available.

Schematic model of the fuel cycle for a DT fusion reactor used in the present work



$\Lambda$  = tritium breeding ratio

$\dot{N}^-$  = tritium burn rate in the plasma

$I_i$  = tritium inventory in compartment  $i$

$T_i$  = tritium mean residence time in compartment  $i$

$\epsilon_i$  = nonradioactive loss of tritium in compartment  $i$

$\lambda$  = tritium decay constant

$\beta$  = tritium fractional burnup in the plasma

$f_i$  = tritium fractional leakage in compartment  $i$

$\dot{I}_0$  = constant flow rate of tritium recovered from waste, steam, and air processing units

TRITIUM INVENTORY VARIATION WITH TIME  
FOR THE BASE CASE PARAMETER VALUES  
USING  $\beta = 0.05$  and  $t_d = 5$  YR

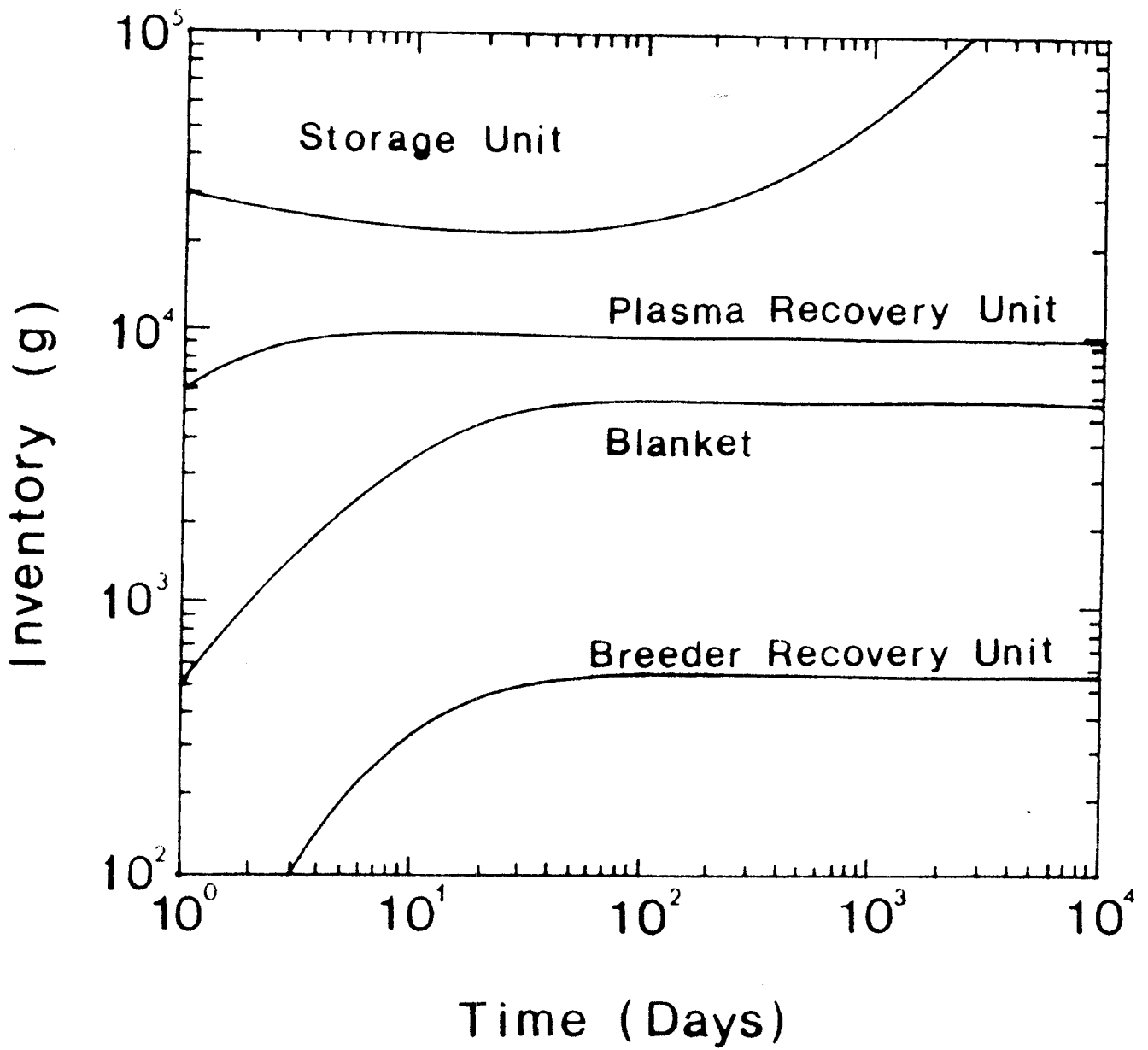


Table 3. Summary of Variation of the Required Breeding Ratio with a Single Change in Each of the Reference Parameters

Parameter (X)	TBR for		Slope	Percentage of Total Slope
	(X <sub>ref</sub> /10)	(X <sub>ref</sub> · 10)		
N̄	1.08(0.0%)*	1.08(0.0%)*	0.0001	0.1
t <sub>d</sub>	1.53(41.7%)	1.04(-3.7%)	-0.0534	24.0
β	1.70(57.4%)	1.02(-5.6%)	-0.0863	38.8
t <sub>r</sub>	1.05(-2.8%)	1.37(26.8%)	0.0322	14.5
ε <sub>2</sub>	1.08(0.0%)	1.09(0.9%)	0.0011	0.5
ε <sub>3</sub>	1.08(0.0%)	1.08(0.0%)	0.0000	0.0
ε <sub>6</sub>	1.06(-1.8%)	1.25(15.7%)	0.0192	8.6
ε <sub>7</sub>	1.08(0.0%)	1.08(0.0%)	0.0000	0.0
ε <sub>8</sub>	1.08(0.0%)	1.08(0.0%)	0.0000	0.0
T <sub>1</sub>	1.07(-0.9%)	1.19(10.2%)	0.0096	4.3
T <sub>2</sub>	1.08(0.0%)	1.09(0.9%)	0.0009	0.4
T <sub>3</sub>	1.08(0.0%)	1.09(0.9%)	0.0015	0.7
T <sub>4</sub>	1.08(0.0%)	1.10(1.9%)	0.0009	0.4
T <sub>6</sub>	1.06(1.8%)	1.22(13.0%)	0.0153	6.9
T <sub>7</sub>	1.08(0.0%)	1.08(0.0%)	0.0002	0.1
T <sub>8</sub>	1.08(0.0%)	1.08(0.0%)	0.0002	0.1
f <sub>c</sub>	1.08(0.0%)	1.09(0.9%)	0.0014	0.6
f <sub>L</sub>	1.08(0.0%)	1.08(0.0%)	0.0004	0.2
f <sub>F</sub>	1.08(0.0%)	1.08(0.0%)	<u>0.0004</u>	<u>0.2</u>
			∑ = 0.2231	∑ = 100.0

\* Values in parenthesis represent the relative change of TBR with respect to a reference TBR of 1.08.



REQUIRED TBR IS FOUND TO BE  
STRONGLY DEPENDENT ON SIX KEY PARAMETERS

$\beta$  = tritium fractional burnup in plasma

$t_d$  = doubling time

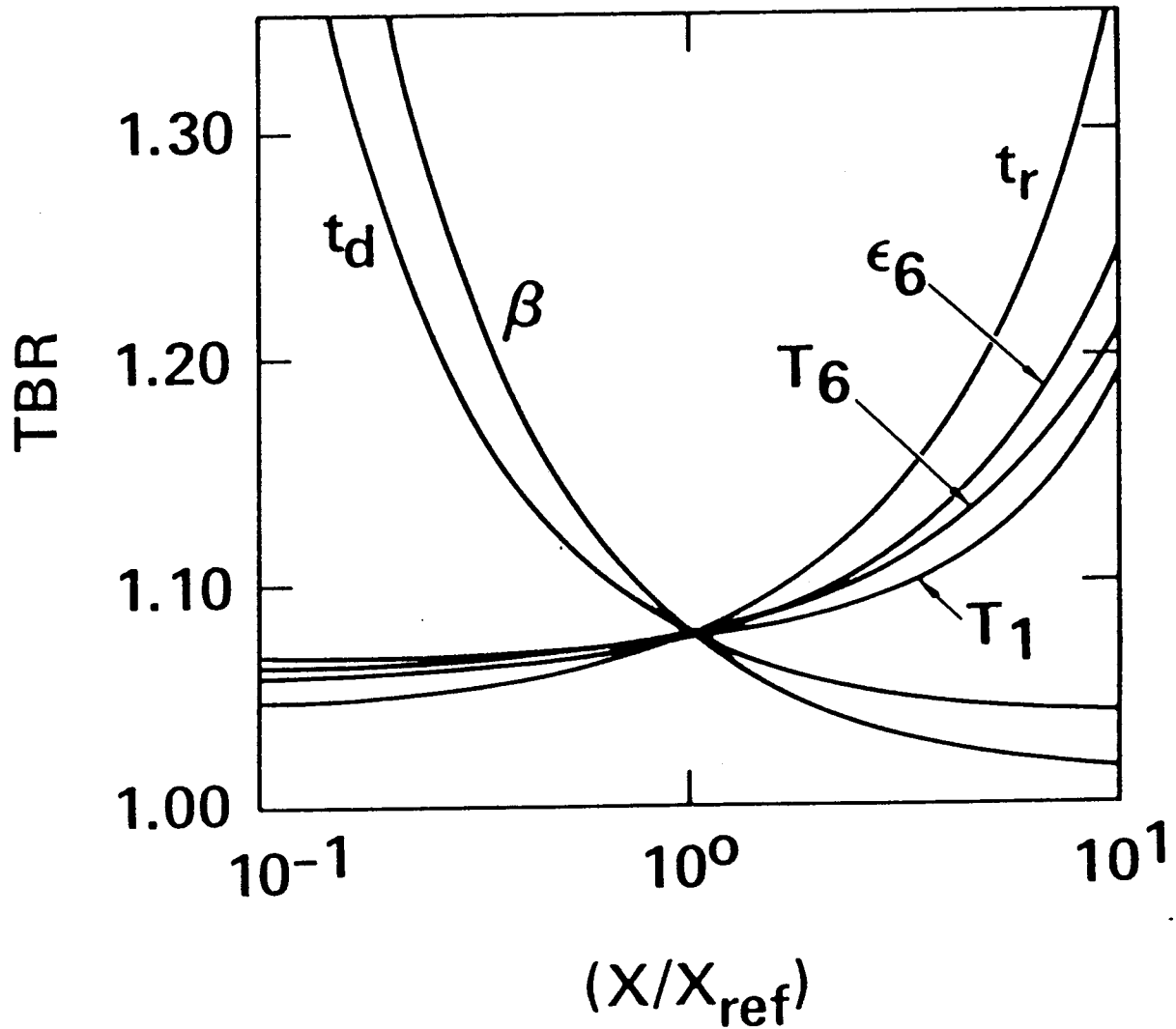
$T_1$  = tritium mean residence time in blanket

$T_6$  = tritium mean residence time in plasma exhaust  
processing

$t_r$  = number of days of tritium reserve

$\epsilon_6$  = tritium extraction inefficiency in plasma exhaust  
processing

## Dependence of Required TBR on Plasma, Engineering Parameters



Reference Case ( $X_{ref}$ )

$$\beta = 5\%$$

$$T_1 = 10d$$

$$t_r = 2d$$

$$t_d = 5y$$

$$T_6 = 1d$$

$$\epsilon_6 = 0.1\%$$

## METHODS FOR CALCULATING $\Delta_G$

### Problem

- We know from mathematical models how to calculate

$$\Lambda_r = f(x_1, x_2, \dots, x_n)$$

where  $x_i$  = reactor parameter

- $x_i$  can not be defined now; we can only estimate a range

### Approach

- Estimate expected values of  $x_i$ 's
- Calculate  $\Lambda_r^0 = f(x_1^0, x_2^0, \dots, x_n^0)$   
 $x_i^0$  = reference reactor parameter
- Define
$$\Lambda_r = \Lambda_r^0 + \Delta_G$$
- Develop probability distribution for  $x_i$   
around  $x_i^0$  (Key problem)
- Estimate  $\Delta_G$

**Log-Normal Probability Distributions  
Used as Weighting Functions, Superimposed  
on the Variation of the Breeding Ratio with Doubling Time.**

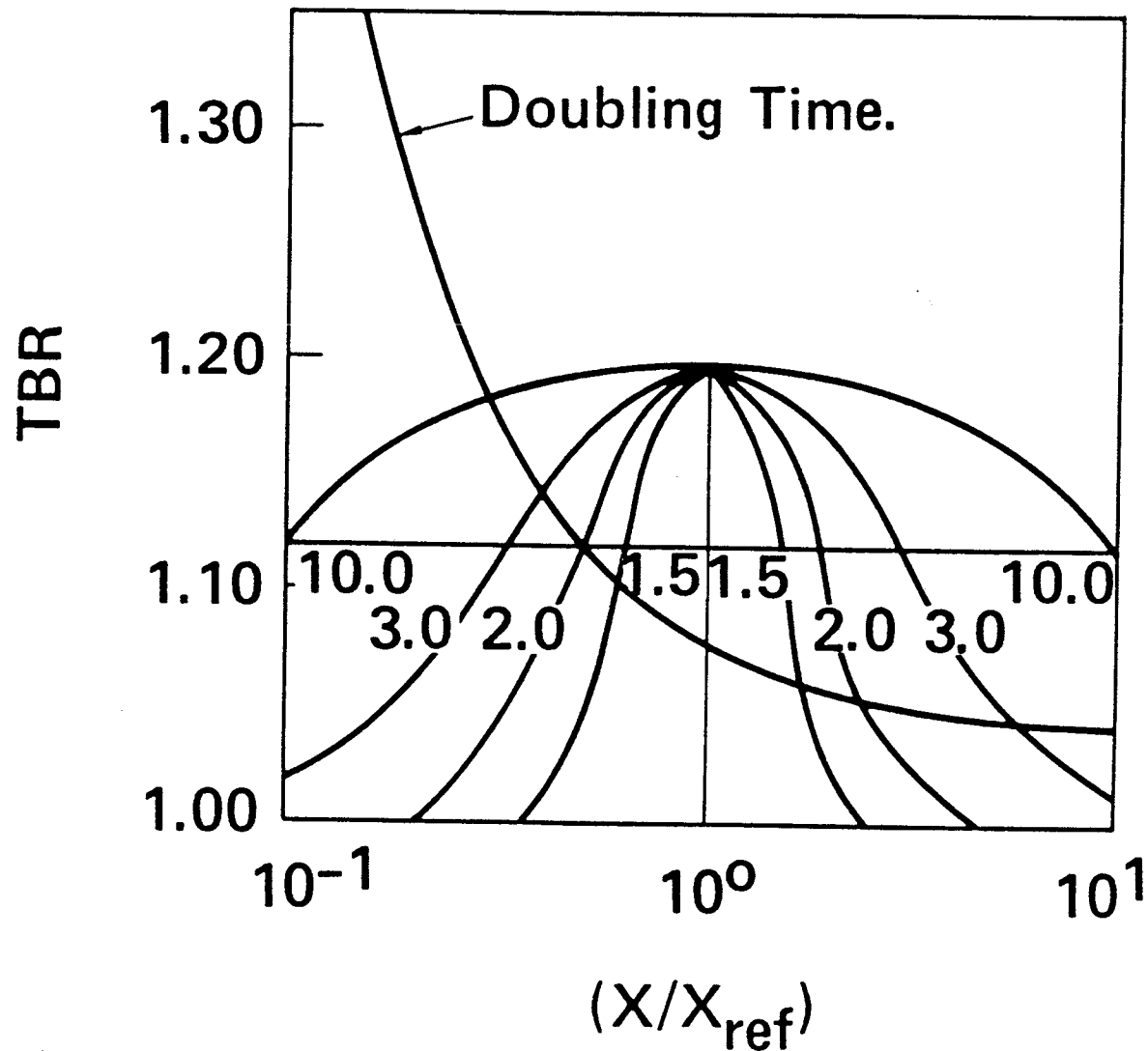


Table 21. Breeding Ratio Weighted for Log-Normal Distribution of Parameter, x

Parameter, x (Base Case Value)	Ref. Case $\sigma_g = 1.0$	$\sigma_g = 1.5$	$\sigma_g = 2.0$	$\sigma_g = 2.5$	$\sigma_g = 3.0$	$\sigma_g = 5.0$
Doubling time, $t_\alpha$ (5 yr.)	1.077	$\frac{1.081}{1.081}$	$\frac{1.089}{1.089}$	$\frac{1.102}{1.100}$	$\frac{1.118}{1.109}$	$\frac{1.334}{1.111}$
Burn fraction, $\beta$ (0.05)	1.077	$\frac{1.083}{1.083}$	$\frac{1.096}{1.096}$	$\frac{1.113}{1.111}$	$\frac{1.133}{1.122}$	$\frac{1.248}{1.123}$
Days of T-reserve, $t_r$ (2 d)	1.077	$\frac{1.079}{1.079}$	$\frac{1.085}{2.085}$	$\frac{1.093}{1.092}$	$\frac{1.102}{1.097}$	$\frac{1.155}{1.097}$
Extraction inefficiency in Plasma Exhaust Processing (0.001)	1.077	$\frac{1.078}{1.078}$	$\frac{1.082}{1.082}$	$\frac{1.086}{1.086}$	$\frac{1.092}{1.089}$	$\frac{1.121}{1.089}$
Residence time in Plasma Exhaust Processing, $T_6$ (1 d)	1.077	$\frac{1.078}{1.078}$	$\frac{1.081}{1.081}$	$\frac{1.084}{1.084}$	$\frac{1.089}{1.086}$	$\frac{1.117}{1.086}$
Blanket residence time, $T_1$ (10 d)	1.077	$\frac{1.078}{1.078}$	$\frac{1.080}{1.080}$	$\frac{1.082}{1.082}$	$\frac{1.086}{1.084}$	$\frac{1.123}{1.083}$

A: Top breeding ratio value is for log-normal parameter x weighting with no boundaries on x value. Bottom breeding ratio value is for log-normal parameter x weighting and defined boundaries on x; in this case set equal to 0.1x (base case value) and 10x (base case value).

Table 22. Log Normal Weighted Mean Breeding Ratio and Corresponding Uncertainty with Selected Values of Standard Deviation

Parameters (x)	$x_g^a$	Selected <sup>b</sup> $\sigma_g$	$\bar{\lambda}$	$\delta_{Gi}$
Doubling time	5 yr	2	1.089	0.011
Burn fraction	.05	3	1.122	0.042
Days of T reserve	2 d	2	1.085	0.007
Extraction inefficiency in plasma exhaust processing	.001	5	1.089	0.011
Residence time in plasma exhaust processing	1 d	2	1.081	0.004
Blanket residence time	10 d	3	1.084	0.006
All other parameters	-	-	1.09	0.012

<sup>a</sup>Reference value for  $x_i$ .

<sup>b</sup>Weighted over range between 0.1 of reference value and ten times the reference value.

REQUIRED BREEDING RATIO UNCERTAINTY  
(95% CONFIDENCE LEVEL)

Parameter	$x_g$	$\sigma_g$	$\Lambda_{ex,i}$	$\Delta_{Gi}$ (%)
Doubling time	5 yr	2	1.120	4
Burn fraction	.05	2.5	1.18	9.6
Days of T reserve	2 d	2	1.108	3
Plasma recovery loss fraction	0.001	5	1.153	7
Plasma recovery time	1 d	2	1.092	1.4
Blanket inventory	5 kg	3	1.097	2

## ACHIEVABLE TBR

- Problem - We cannot predict precisely  $\Lambda_a$  because:
  - We do not know the exact specifications of what to build
  - For given reactor specifications, we cannot predict precisely the performance
- We can only calculate a TBR for a reference system with assumptions about its specifications

$$\Lambda_a = \Lambda_c - \sqrt{\Delta_s^2 + \Delta_p^2}$$



$\Delta_c$  = TBR calculated (the best we know how today, 3D, etc.) for a specified blanket in a specified reactor

$\Delta_s$  = Uncertainty associated with system definition [changes in calculated TBR resulting from changes in the reference reactor system (e.g., reference reactor system has limiter and reactor to be built could have a divertor)]

$\Delta_p$  = Uncertainties in predicting TBR for a given system

$$\Delta_p = \sqrt{\Delta_m^2 + \Delta_d^2 + \Delta_c^2}$$

$\Delta_m$  = Uncertainties associated with geometric modeling

$\Delta_d$  = Uncertainties associated with nuclear data

$\Delta_c$  = Uncertainties associated with calculational methods

## TYPES OF UNCERTAINTIES IN PREDICTING ACHIEVABLE TBR

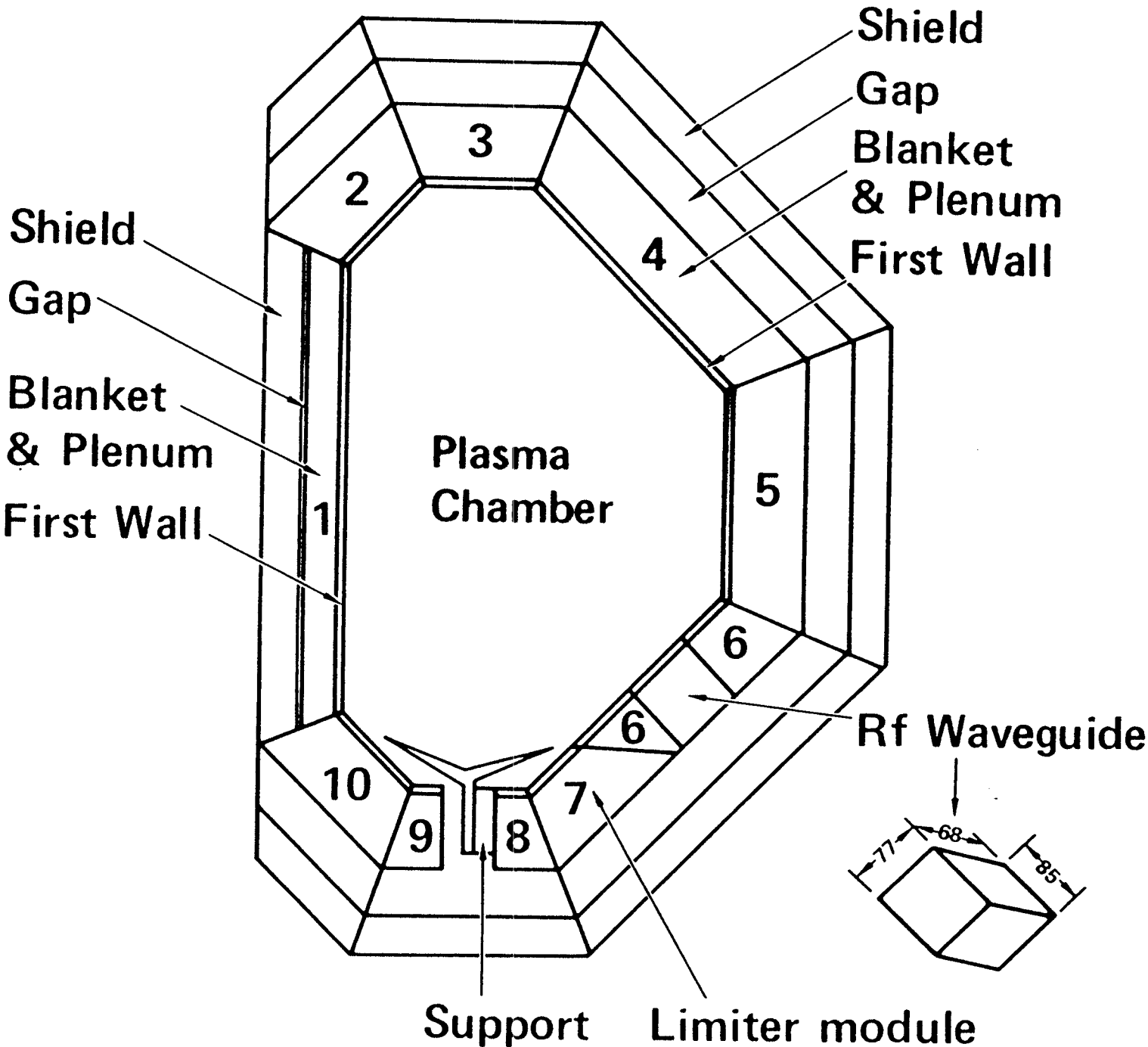
### Uncertainties Associated with System Definition ( $\Delta_S$ )

- First Wall/Blanket Definition
  - Configuration details, structure, coolant, manifolds, form and porosity of solid breeders, thermophysical property variations, etc.
  
- Reactor Definition
  - Technology choices (type of rf vs. neutral beams, limiter vs. divertor, etc.)
  
  - Requirements and specifications for specific technology choices (e.g., size and configuration of penetrations for limiter, material choices for limiter)
  
  - Presence of yet undefined components (e.g., penetrations for diagnostics and fueling, I&C)
  
  - Possible need for components to satisfy yet undefined requirements (e.g., passive copper coils in the blanket for plasma stabilization, sector to sector electrical joints, etc.)

$\Delta_p$  = UNCERTAINTIES ASSOCIATED WITH PREDICTING  
TBR FOR A GIVEN SYSTEM

- Approximations in Geometrical Modeling ( $\Delta_m$ )
  - Approximating engineering 3D surfaces and volumes by traditional mathematically convenient shapes (intersection of cones, cylinders, spheres, cubes, etc.)
  - Approximating discrete by continuous geometric zones
  - Approximating the details of heterogeneity
  
- Nuclear Data ( $\Delta_d$ )
  - Uncertainties in basic nuclear data
  - Approximations in data processing
  - Approximations in final data libraries (number of energy groups, weighting functions, etc.)
  
- Computational Methods ( $\Delta_c$ )
  - Inherent in methods and codes
  - Introduced by analyst (e.g., order of  $S_n$ ,  $P_n$ , etc.)

# Vertical Cross Section of Reference Tokamak Reactor



**UNCERTAINTIES IN ACHIEVABLE BREEDING RATIO  
DUE TO UNCERTAINTIES IN SYSTEM DEFINITION**

Type of Change	Change in TBR (%)
No inboard blanket	14
Limiter: Non-breeding limiter module Doubling limiter duct width Strong absorber coating  Divertor replaces limiter	6 2 4  7
Other penetrations: Auxiliary heating Fueling, diagnostics, etc.	1 1
Other materials in blanket (e.g., passive copper coils)	3
Blanket first wall specifica- tion details (configuration, structure, coolant, manifolds)	2

$\Delta_d$ , ESTIMATE OF UNCERTAINTY IN TBR DUE TO  
UNCERTAINTIES IN NUCLEAR DATA

Blanket Concept	$\Delta_d$ (%)
Li/Li/HT9	5.5
LiPb/LiPb/V	4.4
Li/Li/V	6
Li <sub>2</sub> O/He/HT9	4.9
LiAlO <sub>2</sub> /H <sub>2</sub> O/HT9/Be	2.1

ACHIEVABLE AND REQUIRED TRITIUM BREEDING RATIOS  
AND UNCERTAINTIES FOR LEADING BLANKETS IN TOKAMAKS

Concept	Achievable $\Lambda_a$		Required $\Lambda_r$		$\epsilon = \Lambda_a - \Lambda_r$
	$\Delta_c$	$\Delta_a$	$1 + G_0$	$\Delta_g$	
LiAlO <sub>2</sub> /DS/HT9/Be	1.24	0.22	1.077	0.143	-0.20
LiPb/LiPb/V	1.30	0.24	1.072	0.142	-0.15
Li/Li/V	1.28	0.24	1.072	0.142	-0.17
Li <sub>2</sub> O/He/HT9	1.11	0.21	1.077	0.143	-1.32
LiAlO <sub>2</sub> /He/HT9/Be	1.04	0.19	1.077	0.143	-0.37
Li/He/HT9	1.16	0.22	1.072	0.142	-0.27
LiAlO <sub>2</sub> /H <sub>2</sub> O/HT9/Be	1.16	0.21	1.077	0.143	-0.27

Table 25.  
Limiting Values (Maximum or Minimum) for Key Fusion Reactor Parameters  
that Must Be Achieved to Keep the Required Tritium Breeding Ratio,  $\Lambda_T$ , at 1.05

Parameter	Parameter Limit							
	a	b	c	d	e	f	g	h
Minimum doubling time ( $t_d$ ), yr	20	5	5	1.5	5	5	5	5
Minimum fractional burnup in plasma ( $\beta$ ), %	5	8	5	50	40	21	23	50
Maximum days of tritium reserve ( $t_r$ ), days	2	2	0.2	2	20	2	2	2
Maximum extraction inefficiency in plasma exhaust processing ( $\epsilon_6$ ), %	0.1	0.1	0.1	0.1	0.1	0.5	0.1	0.1
Maximum residence time in plasma exhaust processing ( $T_6$ ), days	1	1	1	1	1	1	10	1
Maximum blanket tritium inventory <sup>a</sup> ( $I_B$ ), kg	5	5	5	5	5	5	5	22

<sup>a</sup>Proportional to blanket mean residence time,  $T_1$ .



Table 26.  
 Limiting Values (Maximum or Minimum) for Key Fusion Reactor Parameters  
 that Must Be Achieved to Keep the Required Tritium Breeding Ratio,  $\Lambda_T$ , at 1.1

Parameter	Parameter Limit									
	a	b	c	d	e	f	g	h	i	j
Minimum doubling time ( $t_d$ ), yr	3.5	5	5	0.75	1.5	5	5	5	5	5
Minimum fractional burnup in plasma ( $\beta$ ), %	5	4	5	50	5	20	12	12	50	5
Maximum days of tritium reserve ( $t_r$ ), days	2	2	3	2	0.2	20	2	2	2	2
Maximum extraction inefficiency in plasma exhaust processing ( $\epsilon_6$ ), %	0.1	0.1	0.1	0.1	0.1	0.1	0.5	0.1	0.1	0.1
Maximum residence time in plasma exhaust processing ( $T_6$ ), days	1	1	1	1	1	1	1	10	1	1
Maximum blanket tritium inventory <sup>a</sup> ( $I_B$ ), kg	5	5	5	5	5	5	5	5	45	22

<sup>a</sup>Proportional to blanket mean residence time,  $T_1$ .